Here are some comments from me, before others' work:

3. This was mostly good. We had anticipated starting with GM = 4 x 10^14 for the earth and using 15% of that. I checked current reported values and they seem to indicate that Mars is actually closer to 10.8% of the Earth, not 15%. Please do not use this for planning your mission to Mars.

4. - 5. were mostly good also. To be clear, I checked data here also. The mean radius of the Earth is about 6.371x10^6m. Rounding that to 6.4 and adding 300 km gives the book number of 6.7 x 10^6 m for distance from the centre of the Earth for the rocket.

8. This was the trickiest one. Most importantly, this is a mathematics class so I need a mathematical justification for your answer. Quoting physics formulae or saying "make this negative" or "use the other value" does not suffice. Both rocket-firings, the beginning and the second should actually increase the speed. So, v1 < 7785 and v2 < 7560. (It is true that we could have computed these circular speeds from the given information. I did, and they checked.) Please read the exemplar and see how the steps follow. A summary is: use apogee and perigee to solve for eccentricity and gamma. From gamma, find K. Once you know K, you can easily find v from r, since at apogee and perigee r and v are perpendicular.

7. There needs to be comprehensible work for computing your line integrals. I like to see pullbacks, but that isn't needed. I do think actual integration _is_ needed, because it isn't a constant form.

16. The best way is demonstrated in the exemplar, there are others. Notice the question says "where along this segment". A t-value doesn't answer that, we need ranges of points on the segment itself.

3.4

③ $r = 3300$ km, $M_m = 0.15\, m_e$

$GM_e = 4 \cdot 10^{14}\ m^3/s^2 \rightarrow GM_m = 6 \cdot 10^{13}\ m^3/s^2$

Using work done from pages 70-71,

$|\varepsilon| > 1$

$\varepsilon^2 = \sin^2\phi \left(1 - \frac{rv^2}{Gm}\right) + \cos^2\phi$

$v_s \geq \sqrt{\frac{2Gm}{r}}$ → independent of ϕ, "escape speed"

↳ doesn't sound as smart

$v_s \geq \sqrt{\frac{1.2 \cdot 10^{14}}{3.3 \cdot 10^6}}$

$\boxed{v_s \geq 6030.23\ m/s}$

Consider a rocket fired 300 km above earth's surface
6.7×10^6 m from the center.

4.) What velocity must it attained if it is to remain in a circular orbit at this height? What is the period of this orbit?

$V_o = \sqrt{\frac{GM}{R}}$ where R is the Radius of orbit

Let $GM = 4 \times 10^{14}\ m^3/s^2$ $\quad R = 6.7 \times 10^6$

$$V_o = \sqrt{\frac{4 \times 10^{14}}{6.7 \times 10^6}} = \boxed{7726.67\ m/s}$$

$$\text{Period} = \frac{2\pi(6.7 \times 10^6)}{7726.67} = 5448.31 \text{ seconds} = \boxed{\text{90 minutes and 48 seconds}}$$

#5. Speed is 9000 m/s and $\phi = \pi/2$. What are the values of the apogee and the perigee?

Since we know the angle b/w $\vec{r}$ ÷ $\vec{v}$ is denoted by ϕ, then $K = rv\sin\phi$

We know $r = 6.7E6$

thus, $K = (6.7E6)(9000)\sin(\pi/2) = 6.03E10$

By equation (3.41) $r = \frac{K^2}{GM} = \frac{(6.03E10)^2}{(4E14)} = 9.09E6$

Finding ε by using eqn (3.54)

$$\varepsilon = \frac{rv^2}{GM} - 1 = \frac{(6.7E6)(9000)^2}{(4E14)} - 1 = 0.35675$$

thus

$$\text{apogee} = \frac{r}{1-|\varepsilon|} = \frac{9.09E6}{1-.35675} = 1.413E7$$

$$\text{perigee} = \frac{r}{1+|\varepsilon|} = \frac{9.09E6}{1+.35675} = 6.6998E6$$

$$\text{period} = \frac{2\pi}{\sqrt{GM}}a^{3/2} = \frac{2\pi(1.041E7)^{3/2}}{\sqrt{(4E14)}} = 10560.3$$

$$a = \frac{r}{1-\varepsilon^2} = \frac{9.09E6}{1-(.35675)^2} = 1.041E7$$

8) $r_1 = 6.6 \times 10^6$ $v_i = 7785$

$= 7.0 \times 10^6$ $v_1' = ?$

apogee $= 7 \times 10^6$ $v_2 = ?$

$r_f = 7.0 \times 10^6$ $v_f = 7560$

perigee $= 6.6 \times 10^6$

$6.6 \times 10^6 (1+\varepsilon) = \gamma$

$7 \times 10^6 (1-\varepsilon) = \gamma$

$6.6 \times 10^6 (1+\varepsilon) = 7 \times 10^6 (1-\varepsilon)$

$1+\varepsilon = 35/33 (1-\varepsilon)$

$1+\varepsilon = 35/33 - (35/33)\varepsilon$

$1 + (68/33)\varepsilon = 35/33$

$(68/33)\varepsilon = 2/33$

$\varepsilon = .0294$

$K = rv\sin\phi$

$5.21 \times 10^{10} = 6.6 \times 10^6 v(\sin \pi/2)$

$v_1 = 7893.94$ m/s

$K = rv\sin\phi$

$5.21 \times 10^{10} = 7 \times 10^6 v(\sin \pi/2)$

$v_2 = 7442.86$

$\gamma = 6.6 \times 10^6 (1.0294)$

$= 6794040$

$\gamma = \frac{K^2}{GM}$

$6794040 = \frac{K^2}{4 \times 10^{14}}$

$K = 5.21 \times 10^{10}$

$\boxed{\begin{aligned} v_1 &= 7893.94 \text{ m/s} \\ v_2 &= 7442.86 \text{ m/s} \end{aligned}}$

Steps
1) find $\vec{r}(t)$
2) compute r^* (differential form)
↳ replace $x\ y\ z$ from $\vec{r}(t)$
3) solve

4.3 #7) $y\,dx + z\,dy + x\,dz$ pullback

a) from $\underset{\vec{a}}{(0,2,-1)}$ to $\underset{\vec{b}}{(3,0,1)}$

$\vec{r}(t) = \vec{a} + t(\vec{b}-\vec{a})$
$= (0,2,-1) + t(3,-2,2)$
$= (\underbrace{3t}_{x}, \underbrace{2-2t}_{y}, \underbrace{-1+2t}_{z})$

(1) $\int_{\vec{r}^{-1}(\vec{a})}^{\vec{r}^{-1}(b)} \vec{r}^*(y\,dx + z\,dy + x\,dz)$

$\vec{r}^*(y)dx = (2-2t)\cdot 3dt$ $\quad r^*(z)dy = -1+2t(-2dt)$ } combine these in (1)
$r^*(x)dz = 3t \cdot 2dt$

$\int_0^1 6 - \cancel{6t} + 2 - 4t + \cancel{6t}\ dt$

$= \int_0^1 8-4t\ dt$

$= 8t - 2t^2 \Big|_0^1 = 8-2 = \boxed{6}$

b) from $\underset{\vec{a}}{(3,0,1)}$ to $\underset{\vec{b}}{(0,2,-1)}$

step 1 $\vec{r}(t) = (3,0,1) + t(-3,2,-2)$
$= (3-3t, 2t, 1-2t) \rightarrow$ $x = 3-3t$, $y=2t$, $z=1-2t$

step 2 $\int_0^1 r^*(y\,dx + z\,dy + x\,dz)$
$r^*(y)dx = 2t \cdot -3dt$
$r^*(z)dy = (1-2t)\cdot 2dt$
$r^*(x)dz = (3-3t)-2dt$

Step 3 $= \int_0^1 -\cancel{6t} + 2 - 4t - 6 + \cancel{6t}\ dt$
$= \int_0^1 -4-4t\ dt$

$= -4t - 2t^2 \Big|_0^1 = -4-2 = \boxed{-6}$

c) from $\underset{\vec{a}}{(3,1,2)}$ to $\underset{\vec{b}}{(-1,1,1)}$

step 1 $\vec{r}(t) = (3,1,2) + t(-4,0,-1)$ $\quad$ ~~x = 3-4t~~, $y=1$, $z = 2-t$
$= (3-4t, 1, 2-t)$

step 2 $\int_0^1 r^*(y\,dx + z\,dy + x\,dz)$
$r^*(y)dx = 1\cdot -4dt$, $\ r^*(z)dy = \cancel{(2-t)\cdot 0dt}$, $\ r^*(x)dz = (3-4t)\cdot -1dt$

Step 3 $\int_0^1 -4 - 3 + 4t\ dt \rightarrow \int_0^1 -7+4t\ dt \rightarrow -7t + 2t^2 \Big|_0^1 = -7+2 = \boxed{-5}$

7d) form: $y\,dx + z\,dy + x\,dz$
from $(0,2,-1)$ to $(1,3,2)$
$\vec{a}$, $\vec{b}$

step 1 $\vec{r}(t) = (0,2,-1) + t(1,1,3)$ $x = t$
$= (t, 2+t, -1+3t)$ $y = 2+t$
$z = -1+3t$

step 2 $\int_0^1 r^*(y\,dx + z\,dy + x\,dz)$

$r^*(y)dx = (2+t)\,dt$ $r^*(z)dy = (-1+3t)\,dt$ $r^*(x)dz = t \cdot 3dt$

step 3 $= \int_0^1 2+t -1+3t+3t\,dt$

$= \int_0^1 7t+1\,dt \rightarrow 3.5t^2 + t \Big|_0^1 = 3.5+1 = \boxed{4.5}$

e) from $(1,3,2)$ to $(3,0,1)$
$\vec{a}$, $\vec{b}$

step 1 $\vec{r}(t) = (1,3,2) + t(2,-3,-1)$ $x = 1+2t$
$= (1+2t, 3-3t, 2-t)$ $y = 3-3t$
$z = 2-t$

step 2 $\int r^*(y\,dx + z\,dy + x\,dz)$

$r^*(y)dx = (3-3t)\,2dt$ $r^*(z)dy = (2-t) \cdot -3dt$ $r^*(x)dz = (1+2t) \cdot -$

step 3 $\int_0^1 \cancel{6-6t} \cancel{-6} \cancel{+3t} -1-2t\,dt$

$\int_0^1 -1-5t\,dt \rightarrow -t-2.5t^2 \Big|_0^1 = -1-2.5 = \boxed{-3.5}$

16. Fluid flow: $(y-y^2)dx + (xy-x)dy$

$\vec{a} = (2,-2)$ $\vec{b} = (2,4)$ Using $(x,y) = \vec{a} + t(\vec{b}-\vec{a})$

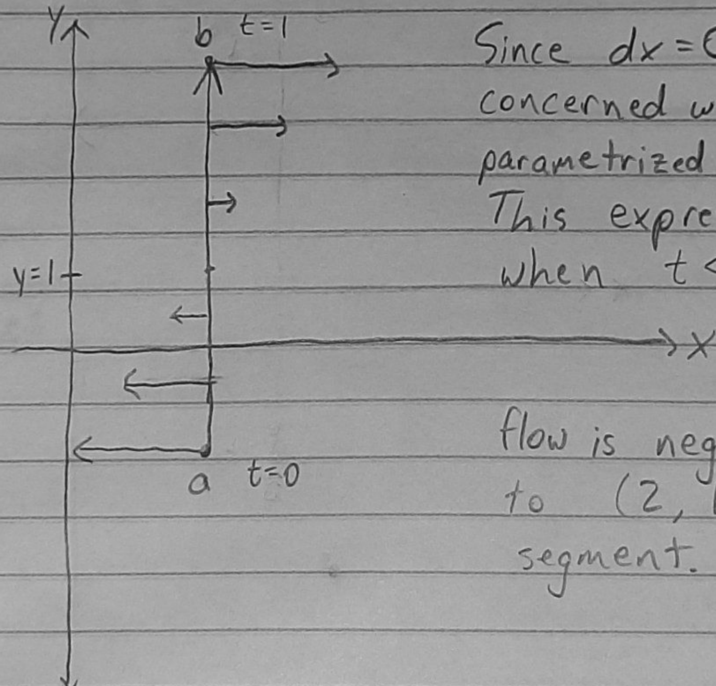
$x = 2$ $dx = 0$

$y = -2 + 6t$ $dy = 6dt$

$$(y-y^2)dx = \left[(-2+6t) - (4 - 12t + 36t^2)\right](0) = 0$$

$$(xy-x)dy = (-4+12t-2)6dt$$

$$\int_0^1 (-36+72t)dt = -36t + 36t^2 \Big|_0^1 = \boxed{0}$$



Since $dx = 0$, we are only concerned with the dy term, parametrized by $-36 + 72t$. This expression is negative when $t < \frac{1}{2}$, so the flow is negative from $(2,-2)$ to $(2,1)$ along the line segment.