The History of Long Division

By: Anna Taylor

Division, at its core, combines multiplication and subtraction for a solution, but its complexity increases with larger numbers. Early mathematicians refined division methods to handle these challenges, finding ways to break down the calculations into manageable chunks. By exploring the evolution of long division through 4 unique examples, we can highlight how different cultures and mathematicians contributed to shaping the division techniques we rely on.

The method of division is made up of a combination of multiplication and subtraction [6]. The idea of division is to consider how many times the divisor can be subtracted from the dividend, creating equal groups of the divisor. With single or double-digit numbers, this is a simple task. However, when working with 5 and 6-digit numbers, the math becomes more complex. To solve this issue, mathematicians started to consider place value, looking to see if they could reduce the size of the calculations, while still accounting for all the digits and the entire value of both numbers.

The first time history captures a long division algorithm in print was somewhere between the 3rd and 5th century, in China, when Sun Zi published his *Sunzi Suanjing*, a mathematics book consisting of methods to multiply, divide, work with fractions, etc [9, 12]. In this text, Zi explains his division procedure using a counting board. In this method, the quotient is placed on top, the dividend is represented in the middle, and the divisor is at the bottom, below the dividend [9]. In this method, the first step is to estimate how many times the divisor fits into the leading digits of the dividend. This estimated quotient digit is recorded in the appropriate position. Next, the divisor is multiplied by the current quotient digit, and the resulting product is subtracted from the corresponding digits of the dividend using rod manipulation on the board.

After this subtraction, the next lower digit of the dividend is brought down, and the process is repeated. The algorithm continues in this digit-by-digit fashion, repeating estimation, multiplication, subtraction, and bringing down the next digit until the entire dividend has been processed. This method is a condensed version of the modern long division algorithm, wiping away the scratch work when manipulating the counting rods[11]. Significantly, we see early work with base-10 and the place-value system, reflecting its importance in our modern methods.

The next remarkable method for solving long division is the "Galley" or "Scratch" method. Although the origin of the algorithm is mostly undetermined, we do know that China and India both started using it around the same time, in 800 AD[1]. The Galley method was used up until the 18th century in many parts of the world, starting in the East, and slowly working its way to the West. It is also known that the Arab mathematician, Al-Khwarizm, used this method of division in his book, *"Kitab al-Jabr wa'l-Muqabala,* highlighting it as an effective way to do calculations [1]. The scratching from its name refers to crossing out digits after using them in calculations. Sometimes, calculations would be done on a sandboard. In these cases, digits were simply wiped away, until only the answer remained. However, the term "Galley" came from the finished product, as connecting lines around your final calculations will turn one's scratch work into a sufficient image of a boat, as seen in Figures 1 and 2 [5, 8].

Figure 1

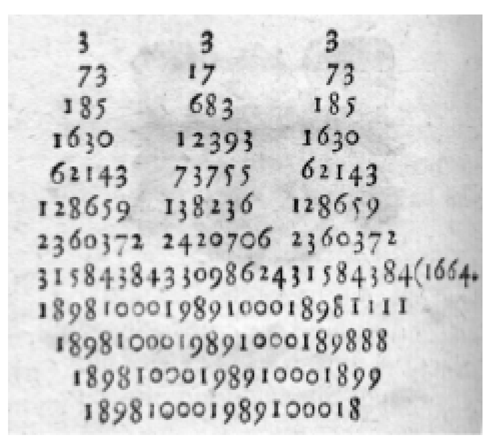

Figure 2

To highlight the significance and steps of this method, I will walk through an easier example, as seen in Figure 3; consider the problem 65284 ÷ 594 [5].

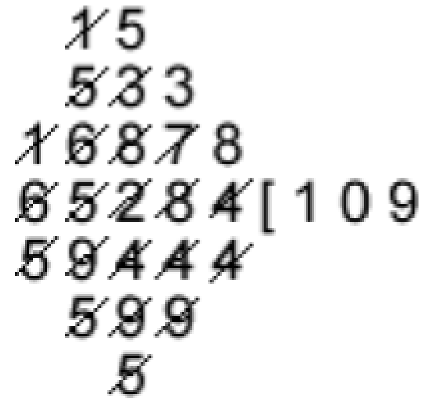

Figure 3

The first step in this method is the setup, with the dividend written first, followed by the divisor underneath it. The dividend and divisor are lined up in columns, starting on the left side and moving to the right. A square bracket or line is then places to the right of the dividend, to keep our quotient digits separate from our working calculations (see Figure 4) [5].

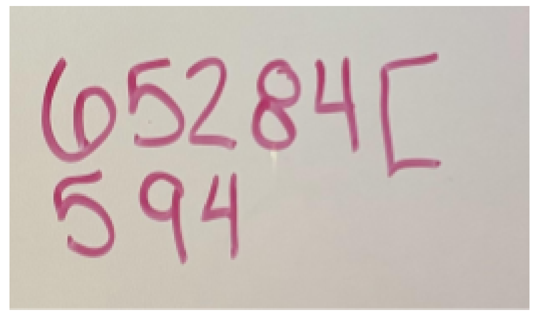

Figure 4

Start the calculation process by considering the whole divisor and seeing how many times it can be subtracted from the dividend, using the fewest digits possible from the dividend, to yield a result of at least one. For this example, we would consider how many times 594 can be subtracted from 652, as anything smaller than these place values will not yield a difference. We conclude that 594 can be subtracted from 652, 1 time, so a 1 is placed on the right side of our bracket in the space for the digits of the quotient.

Next, we multiply the first divisor digit and the first quotient digit together, 1 x 5. The product of that, 5, is then subtracted from the first dividend digit, 6. After calculations, we know 6-5=1, so a 1 digit is placed directly over the 6. This is done to preserve place value. Finally, we can cross out the digits 6 and 5 in the far left place value, as we have just used them for calculations and will not use them again (see Figure 5).

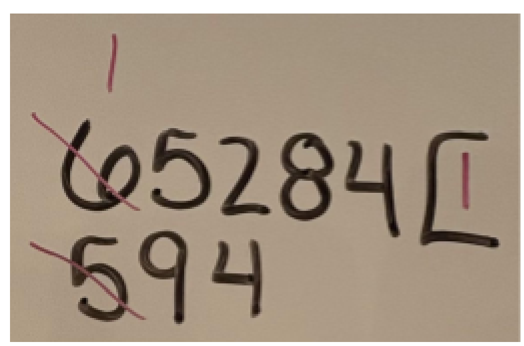

Figure 5

The next step in the algorithm is to consider the second digit in the divisor, 9, the 1 in the quotient, and the new number we just created in the dividant,15, using the previous remainder of 1 in the thousandth place of our dividend. We will do the same steps as above, multiplying the 9 by the 1 in the quotient, 9x1, and subtracting that from our new number of 15. 15-9=6, so 6 will be placed over the 5 in the dividend, keeping it in the correct place value. We then cross out the digits 1 and 5 in the dividend along with the 9 in the divisor, as they were all used in calculations (see Figure 6).

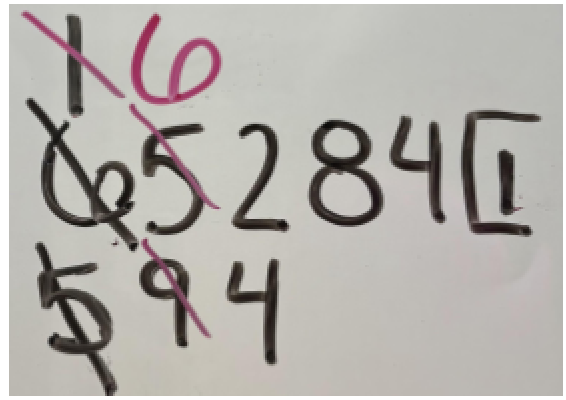

Figure 6

Next, consider the 4 in the divisor, the 1 in the quotient, and the new number, 62, that we just created in the dividend. We will subtract 4x1, or 4, from 62; the result is 58. This 58 is then placed above the 6 and 2 of the dividend, respectively, keeping with place value. We will follow by crossing out the digits 6, 2, and 4, as they have all been used. See Figure 7 to see the first chunk of steps completed.

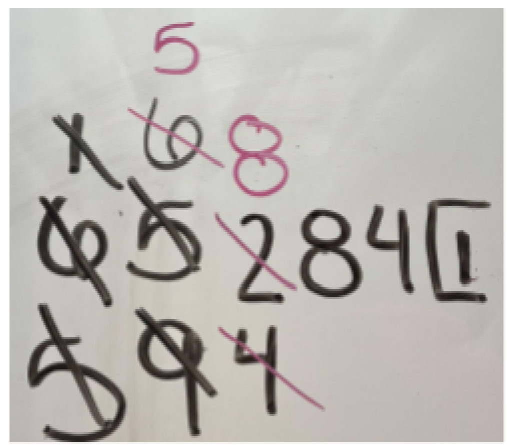

Figure 7

Then, because we no longer have a divisor to divide by, we will write out divisor digits again, shifting one column to the right, as our dividend has shifted one column, or place value, as well. See Figure 8 for the setup for the next set of calculations.

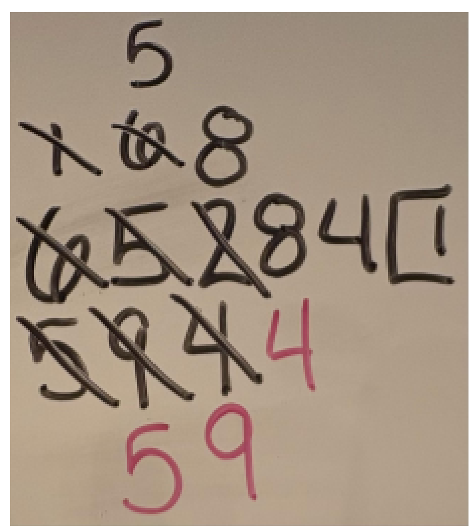

Figure 8

These calculations repeat twice more, with the same steps as above. See figure 9 for the second set of calculations, and figure 10 for the last set of calculations.

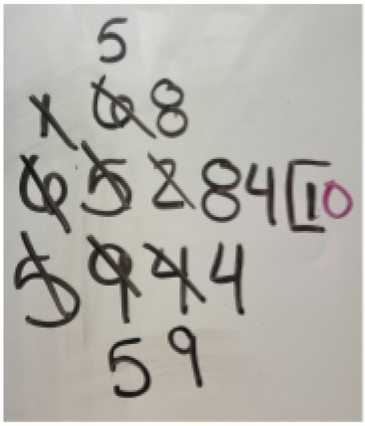

Figure 9

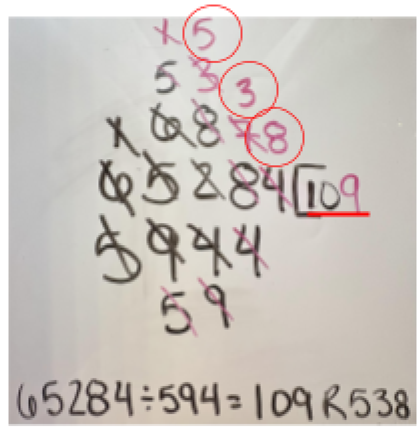

Figure 10

When looking at our work, it is clear to find our quotient in the box to the right, and our remainder in the calculations to the left, as anything that is not crossed out is a remainder. When doing this method, we get the correct answer of 65284 ÷ 594 =109 R 538.

The third division algorithm was found in Fibonacci's *Liber Abaci* in 1202 CE. His book helped bring calculations to the people of the West, providing information and algorithms in a form that common people could understand and replicate. In his approach to division, divisors are represented by their prime factorization, in a fraction composition, written in Arabic notation, which is read from right to left [1]. This algorithm uses prime numbers instead of base-10 place values as a way to reduce the size of the computations.

Consider the example 654 ÷ 45 for Fibonacci's algorithm. The 45 is represented as 5x3x3, or its prime factorization, as 5x3=15 and 15x3=45.

Fibonacci places the divisor, in prime factorization form, on the bottom of a fraction, with an empty spot for the remainder in the numerator, and the dividend to the left of the fraction, separated by a vertical line. Because of this, we will set up our example with 654 on the left, a fraction with nothing in the numerator, and 5 3 3 in the denominator (see Figure 11).

After the initial setup, we will consider just the first factor in the divisor. We will divide 654 by 5, resulting in 130 with a remainder of 4 or R4. The remainder 4 is placed in the numerator, over the 5 in the denominator, which is the digit we used in our calculations, keeping place value (see Figure 12).

Next, we will take the 130 found from the previous computation and divide it by the next digit in the denominator, 3. When we do this, 130 ÷ 3 results in 43 R1. This 1 is then placed in the numerator above the leftmost 3 (see Figure 13).

Finally, following the same steps, we will consider 43 divided by the remaining digit in the denominator, 3. Because 43 ÷ 3 results in 14 R1, the final remainder is placed in the numerator, over the last remaining digit in the denominator, the 3. At this point, the 14, or the final quotient result, is placed on the right side of the prime factorization of the remainder, representing the whole number of the quotient (see Figure 14). The solution to 654 ÷ 45 = 1/3 + 1/9 + 4/45 +14 = 14 +2/8.

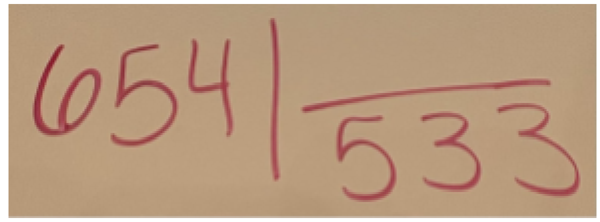

Figure 11

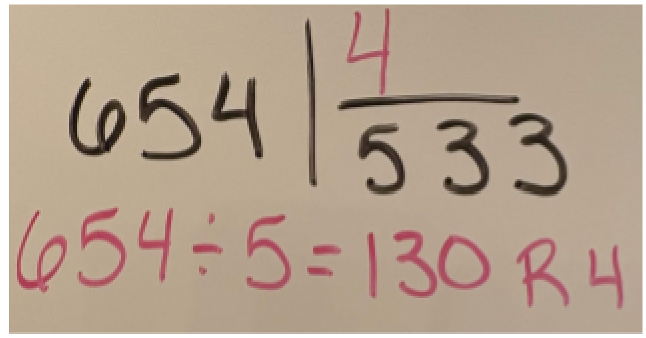

Figure 12

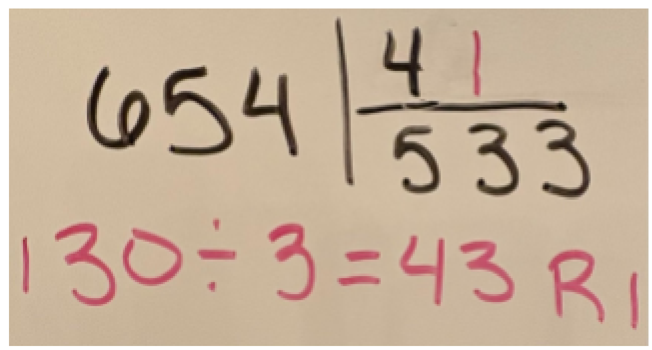

Figure 13

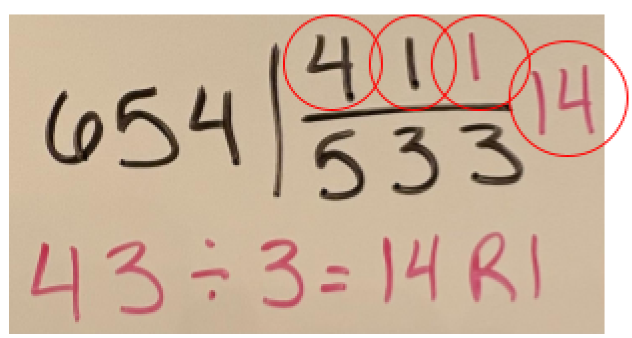

Figure 14

In summary, there was division by each factor of the divisor. We then added up all of the remainders of each divisor and eventually found our overall quotient.

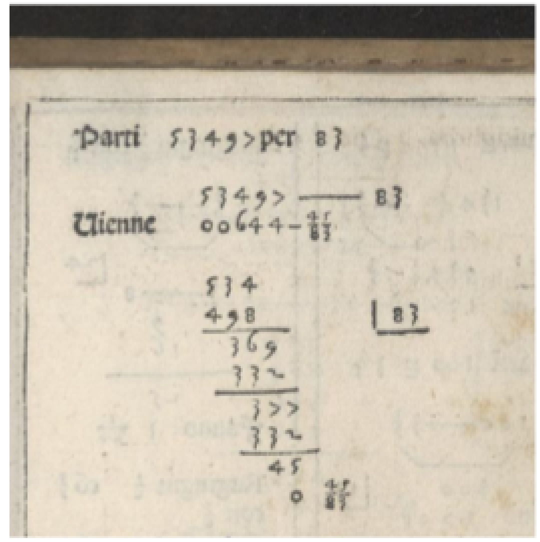

Figure 15

The last major division algorithm to appear in historical text was in Calandri's *Arithmetica,* once referred to as the "Italian method" or "Danda" method [3]. Ironically, this method is extremely similar to the first example shown, using subtraction and multiplication in base-10 to make calculations one place value at a time. The only significant difference is the placement of the scratchwork, as it is kept in the algorithm instead of pushed away, as it would have been on a counting board. Calandri's original example can be seen in Figure 15, with striking similarities to the long division algorithm we use today.

Due to its easy steps, this algorithm is the one most commonly used today, but it is certainly not the first. The history of long division showcases the evolution of mathematical thought and problem-solving across different cultures and time periods. Though the methods have varied, the core principles of multiplication and subtraction have remained constant. Over time, mathematicians refined division techniques to make calculations more efficient and accessible, ultimately shaping the standard algorithm taught today. Understanding this progression not only highlights human ingenuity but also deepens our appreciation for the mathematical foundations that continue to influence modern learning.

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