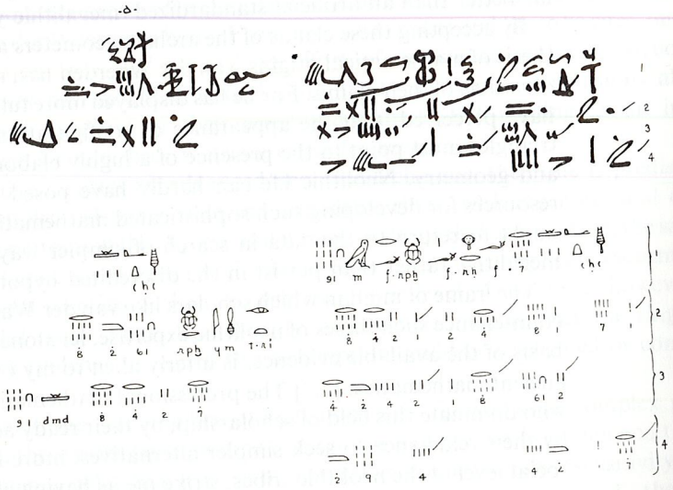
1.D Egyptian Mathematics

For a literate civilization extending over some 4000 years, that of the ancient Egyptians has left disappointingly little evidence of its mathematical attainments. Even though the classical Greeks believed mathematics to have been invented in Egypt (1.D4)—though their accounts are far from unanimous on how this happened—there are now but a handful of papyri and other objects to convey a sense of Egyptian mathematical activity. The largest and best preserved of these is the Rhind papyrus (1.D1, 1.D2), now in the British Museum, a copy made in about 1650 BC of a text from two centuries earlier. A lively picture of one of the contexts in which mathematics was used is provided by a satirical letter (1.D3) from later that millennium (perhaps 1500–1200 BC); the writer adopts a jocular attitude towards his colleague's attempts at quantity surveying. 1.D5–1.D7 are modern commentaries. In 1.D5 the Egyptologist Sir Alan Gardiner explains an initially puzzling feature of Egyptian arithmetic, the Egyptian concept of fraction or part. 1.D6 and 1.D7 are contrasting perceptions of Egyptian mathematics, from the translator of the Rhind papyrus and from a historian of mathematics.

1.D1 Two problems from the Rhind papyrus

(*a*) *Problem 24*



ʿḥʿ	7·f	ḥr·f	ḫpr·fm	19.

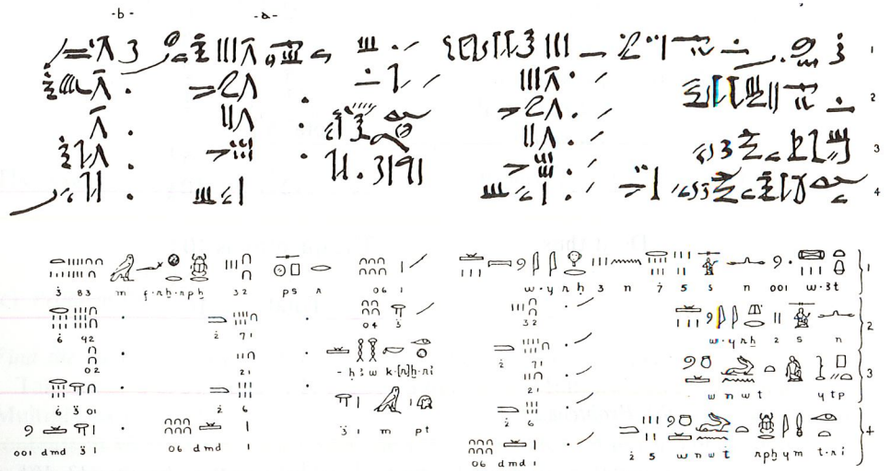
A quantity, $\frac{1}{7}$ of it added to it, becomes it: 19.

\1	7
\7	1

1	8
\2	16
2̇	4
\4̇	2
\8̇	1
\1	2 4̇ 8̇
\2	4 2̇ 4̇
\4	9 2̇

ỉr·t my ḫpr	
The doing as it occurs.	
ʿḥʿ	16 2̇ 8̇
The quantity	
7̇	2 4̇ 8̇
dmd	19.
Total	

(*b*) *Problem 40*



t̓·w 100 n s 5 7̇ n 3 ḥry·w n s 2 ḥry·w pty twnw

Loaves 100 for man 5, $\frac{1}{7}$ of the 3 above to man 2 those below. What is the difference of share?

ỉr·t	my	ḫpr	twnw	5 2̇	ir·ḫ[r]·k	w̓ḥ–tp	m 1 3̇
The doing as it occurs. The difference of share being 5 $\frac{1}{2}$,					Make thou the multiplication:		1 $\frac{2}{3}$
\1		23			r sp 23	ḫpr·ḫr·f m	38 3̇
\1		17 2̇			up to times	becomes it:	
\1		12			“ 17 2̇	“	29 6̇
\1		6 2̇			“ 12	“	20
\1		1			“ 6 2̇	“	10 3̇ 6̇
dmd		60			“ 1	“	1 3̇
Total					dmd 60	dmd	100.
					Total	Total	

\1	60
\3̇	40

1.D2 More problems from the Rhind papyrus

(*a*) *Problem 25*

A quantity and its $\frac{1}{2}$ *added together become 16. What is the quantity?*

Assume 2.

\1	2
\$\frac{1}{2}$	1
Total	3.

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

\1	3
2	6
\4	12
$\frac{2}{3}$	2
\$\frac{1}{3}$	1
Total $5\frac{1}{3}$.	

Do it thus:

	The quantity is $10\frac{2}{3}$
$\frac{1}{2}$	$5\frac{1}{3}$
Total	16.

(*b*) *Problem 31*

A quantity, its $\frac{2}{3}$*, its* $\frac{1}{2}$*, and its* $\frac{1}{7}$*, added together, become 33. What is the quantity?*

Multiply $1\frac{2}{3}\frac{1}{2}\frac{1}{7}$ so as to get 33.

1	$1\frac{2}{3}\frac{1}{2}\frac{1}{7}$
\2	$4\frac{1}{3}\frac{1}{4}\frac{1}{28}$
\4	$9\frac{1}{6}\frac{1}{14}$
\8	$18\frac{1}{3}\frac{1}{7}$
$\frac{1}{2}$	$\frac{1}{2}\frac{1}{3}\frac{1}{4}\frac{1}{14}$
\$\frac{1}{4}$	$\frac{1}{4}\frac{1}{6}\frac{1}{8}\frac{1}{28}$.

Total $14\frac{1}{4}$. $14\frac{1}{4}$ times $1\frac{2}{3}\frac{1}{2}\frac{1}{7}$ makes $32\frac{1}{2}$ plus the small fractions $\frac{1}{7}\frac{1}{8}\frac{1}{14}\frac{1}{28}\frac{1}{28}$. $32\frac{1}{2}$ from 33 leaves the remainder $\frac{1}{2}$ to be made up by these fractions and a further product by a number yet to be determined.

$\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$ $\frac{1}{28}$

taken as parts of 42 are

$$6 \quad 5\tfrac{1}{4} \quad 3 \quad 1\tfrac{1}{2} \quad 1\tfrac{1}{2},$$

making in all $17\frac{1}{4}$, and requiring $3\frac{1}{2}\frac{1}{4}$ more to make 21, $\frac{1}{2}$ of 42.

Take $1\frac{2}{3}\frac{1}{2}\frac{1}{7}$ as applying to 42:

\1	42
\$\frac{2}{3}$	28
\$\frac{1}{2}$	21
\$\frac{1}{7}$	6
Total	97.

That is, $1\frac{2}{3}\frac{1}{2}\frac{1}{7}$ applied to 42 gives 97 in all. $\frac{1}{42}$ of 42, or 1, will be $\frac{1}{97}$ of this, and $3\frac{1}{2}\frac{1}{4}$ will be $3\frac{1}{2}\frac{1}{4}$ times as much. Therefore we multiply $\frac{1}{97}$ by $3\frac{1}{2}\frac{1}{4}$.

\$\frac{1}{97}$	$\frac{1}{42}$ or 1 as a part of 42
\$\frac{1}{56}\frac{1}{679}\frac{1}{776}$	$\frac{1}{21}$ or 2 as a part of 42
\$\frac{1}{194}$	$\frac{1}{84}$ or $\frac{1}{2}$ as a part of 42
\$\frac{1}{388}$	$\frac{1}{168}$ or $\frac{1}{4}$ as a part of 42.

The total is $14\frac{1}{4}\frac{1}{56}\frac{1}{97}\frac{1}{194}\frac{1}{388}\frac{1}{679}\frac{1}{776}$, which multiplied by $1\frac{2}{3}\frac{1}{2}\frac{1}{7}$ makes 33.

(*c*) *Problem 41*

Find the volume of a cylindrical granary of diameter 9 and height 10.

Take away $\frac{1}{9}$ of 9, namely, 1; the remainder is 8. Multiply 8 times 8; it makes 64. Multiply 64 times 10; it makes 640 cubed cubits. Add $\frac{1}{2}$ of it to it; it makes 960, its contents in *khar*. Take $\frac{1}{20}$ of 960, namely 48. 4800 *hekat* of grain will go into it.

Method of working out:

1	8
2	16
4	32
\8	64.

1	64
\10	640
\$\frac{1}{2}$	320
Total	960
$\frac{1}{10}$	96
\$\frac{1}{20}$	48.

(*d*) *Problem 42*

Find the volume of a cylindrical granary of diameter 10 and height 10.

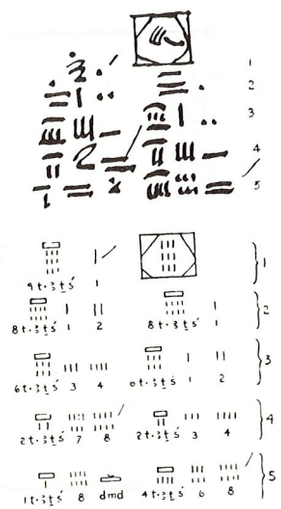
Take away $\frac{1}{9}$ of 10, namely $1\frac{1}{9}$; the remainder is $8\frac{2}{3}\frac{1}{6}\frac{1}{18}$. Multiply $8\frac{2}{3}\frac{1}{6}\frac{1}{18}$ times $8\frac{2}{3}\frac{1}{6}\frac{1}{18}$; it makes $79\frac{1}{108}\frac{1}{324}$. Multiply $79\frac{1}{108}\frac{1}{324}$ times 10; it makes $790\frac{1}{18}\frac{1}{27}\frac{1}{54}\frac{1}{81}$ cubed cubits. Add $\frac{1}{2}$ of it to it; it makes $1185\frac{1}{6}\frac{1}{54}$, its contents in *khar*. $\frac{1}{20}$ of this is $59\frac{1}{4}\frac{1}{108}$. $59\frac{1}{4}\frac{1}{108}$ times 100 *hekat* of grain will go into it.

Method of working out:

1	$8\frac{2}{3}\frac{1}{6}\frac{1}{18}$
2	$17\frac{2}{3}\frac{1}{9}$
4	$35\frac{1}{2}\frac{1}{18}$
\8	$71\frac{1}{9}$
\$\frac{2}{3}$	$5\frac{2}{3}\frac{1}{6}\frac{1}{18}\frac{1}{27}$
$\frac{1}{3}$	$2\frac{2}{3}\frac{1}{6}\frac{1}{12}\frac{1}{36}\frac{1}{54}$
\$\frac{1}{6}$	$1\frac{1}{3}\frac{1}{12}\frac{1}{24}\frac{1}{72}\frac{1}{108}$
\$\frac{1}{18}$	$\frac{1}{3}\frac{1}{9}\frac{1}{27}\frac{1}{108}\frac{1}{324}$
Total	$79\frac{1}{108}\frac{1}{324}$.

1	$79\frac{1}{108}\frac{1}{324}$
10	$790\frac{1}{18}\frac{1}{27}\frac{1}{54}\frac{1}{81}$
$\frac{1}{2}$	$395\frac{1}{36}\frac{1}{54}\frac{1}{108}\frac{1}{162}$
Total	$1185\frac{1}{6}\frac{1}{54}$
$\frac{1}{10}$	$118\frac{1}{2}\frac{1}{54}$
\$\frac{1}{20}$	$59\frac{1}{4}\frac{1}{108}$.

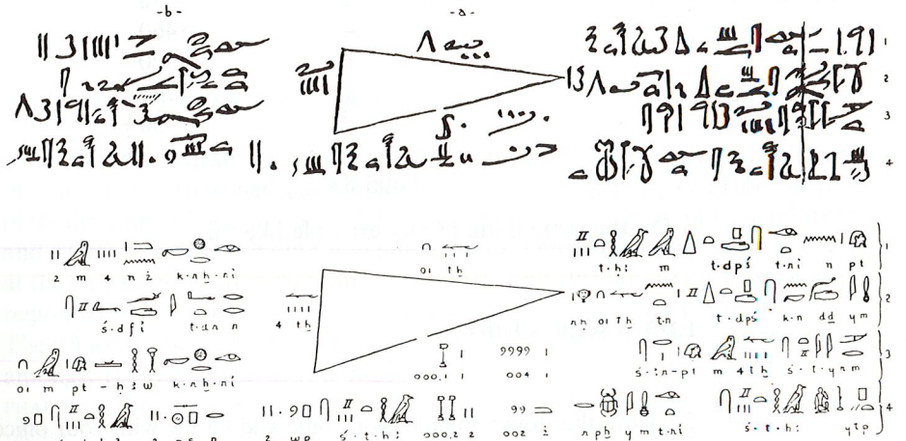
(*e*) *Problem 48*



Compare the area of a circle and of its circumscribing square.

The circle of diameter 9.		The square of side 9.	
1	8 *setat*	\1	9 *setat*
2	16 *setat*	2	18 *setat*
4	32 *setat*	4	36 *setat*
\8	64 *setat*	\8	72 *setat*
		Total	81 *setat*

(*f*) *Problem 51*



Example of a triangle of land. Suppose it is said to thee, What is the area of a triangle of side 10 khet *and of base 4* khet?

Do it thus:

1	400
$\frac{1}{2}$	200
1	1000
2	2000.

Its area is 20 *setat*.

Take $\frac{1}{2}$ of 4, in order to get its rectangle. Multiply 10 times 2; this is its area.

(*g*) *Problem 65*

Example of dividing 100 loaves among 10 men, including a boatman, a foreman, and a door-keeper, who receive double portions. What is the share of each?

The working out. Add to the number of the men 3 for those with double portions; it makes 13. Multiply 13 so as to get 100; the result is $7\frac{2}{3}\frac{1}{39}$. This then is the ration for seven of the men, the boatman, the foreman, and the door-keeper receiving double portions.

For proof we add $7\ \frac{2}{3}\ \frac{1}{39}$ taken 7 times and $15\ \frac{1}{3}\ \frac{1}{26}\ \frac{1}{78}$ taken 3 times for the boatman, the foreman, and the door-keeper. The total is 100.

(*h*) *Problem 66*

If 10 hekat *of fat is given out for a year, what is the amount used in a day?*

The working out. Reduce the 10 *hekat* to *ro*; it makes 3200. Reduce the year to days; it makes 365. Get 3200 by operating on 365. The result is $8\ \frac{2}{3}\ \frac{1}{10}\ \frac{1}{2190}$. This makes for a day $\frac{1}{64}$ *hekat* $3\ \frac{2}{3}\ \frac{1}{10}\ \frac{1}{2190}$ *ro*.

Do it thus:

1	365
2	730
4	1460
8	2920
$\frac{2}{3}$	$243\ \frac{1}{3}$
$\frac{1}{10}$	$36\ \frac{1}{2}$
$\frac{1}{2190}$	$\frac{1}{6}$

Total $8\ \frac{2}{3}\ \frac{1}{10}\ \frac{1}{2190}$.

Do the same thing in any example like this.

1.D3 A scribe's letter

[...] Another topic. Behold, you come and fill me with your office. I will cause you to know how matters stand with you, when you say 'I am the scribe who issues commands to the army'.

You are given a lake to dig. You come to me to inquire concerning the rations for the soldiers, and you say 'reckon it out'. You are deserting your office, and the task of teaching you to perform it falls on my shoulders.

Come, that I may tell you more than you have said: I cause you to be abashed [?] when I disclose to you a command of your lord, you, who are his Royal Scribe, when you are led beneath the window [of the palace, where the king issues orders] in respect of any goodly [?] work, when the mountains are disgorging great monuments for Horus [the king], the lord of the Two Lands [Upper and Lower Egypt]. For see, you are the clever scribe who is at the head of the troops. A [building-] ramp is to be constructed, 730 cubits long, 55 cubits wide, containing 120 compartments, and filled with reeds and beams; 60 cubits high at its summit, 30 cubits in the middle, with a batter of twice 15 cubits and its pavement 5 cubits. The quantity of bricks needed for it is asked of the generals, and the scribes are all asked together, without one of them knowing anything. They all put their trust in you and say, 'You are the clever scribe, my friend! Decide for us quickly! Behold your name is famous; let none be found in this place to magnify the other thirty! Do not let it be said of you that there are things which even you do not know. Answer us how many bricks are needed for it?'

See, its measurements [?] are before you. Each one of its compartments is 30 cubits and is 7 cubits broad.