survived are inscribed stone monuments and hieroglyphic codices. The most famous codex is the (presumably) 12th-century Dresden Codex (named for the European library in which it is housed). The Maya did not leave behind a single treatise on mathematical or astronomical theories and methods nor on mathematical proofs or algorithms. The cumbersome system of writing with glyphs must have hindered, if not prohibited, the preparation and transmission of such mathematical disquisitions. Not a single name of the contributors to Maya mathematics, astronomy, or calendrics has survived. Thus the men of Maya science remain totally anonymous.

The achievements of the Classic Maya in mathematics and calendrical astronomy were remarkable nevertheless. Like the ancient Babylonian scribes, their learned men had a number system with a place-value notation that was used in commercial records, taxes or levies of tribute, census, mensuration, eclipse-possible records in astronomy, and other governmental or religious functions. Their place-value system, which was perhaps seven centuries old even when the Classic period began in the 3rd century, had a symbol for zero (an ornate shell). The zero symbol was used as we use it today. The Maya numeration was vigesimal, having bar-and-dot numerals for the numbers one through twenty. A repetition of dots represented units up to four; bars were for multiples of five; and combinations of bars and dots represented intervening numbers. The choice of base-twenty perhaps evolved from counting on both fingers and toes. The word for twenty in the different Maya languages is often "man," referring to the totality of these digits. The Maya number system also had a decimal substratum. Besides having words for *twenty* or *score*, the Maya had words for multiples of twenty and powers of twenty. The names of Maya numerical units go up to 20^6, while their calendar cycles involve numerals as large as 360×20^{12} units. Historical and archaeological records suggest that the Classic Maya had the most highly developed number system in pre-Columbian America. They lacked a fractional arithmetic, however. In calendrical astronomy the Maya priests developed a rather accurate "calendar year" consisting of eighteen named months of twenty days each and a residue period of five days.

52. From *The Ancient Maya**

SYLVANUS GRISWOLD MORLEY

H. G. Wells in his *Outline of History* says that the invention of a graphic system is the true measure of civilization, and Edward Gibbon, in his *Decline and Fall of the Roman Empire,* claims that the use of letters is the principal characteristic which distinguishes civilized people from savages. By such standards, the Maya were the most civilized people of the New World in pre-Columbian times, since they alone originated a system of writing.

THE DEVELOPMENT OF WRITING

Writing seems to have passed everywhere through three stages of development:

1. *Pictorial or representative writing,* wherein a picture of the idea is portrayed. Thus a deer hunt is represented by the picture of a deer and a man throwing a spear at it. This is called pictographic writing.

2. *Ideographic writing,* wherein characters stand for ideas rather than representing pictures of them. In ideographic writing the characters employed are usually little more than conventionalized symbols. In Chinese writing, the ideograph for "trouble" is the conventionalized symbol for a woman, repeated twice, standing under a gate.

3. *Phonetic writing,* wherein the characters have lost all resemblance to the objects they originally portrayed, and denote only sounds. Phonetic writing may be further divided into *(a)* syllabic writing, in which each character stands for a syllable, and *(b)* alphabetic writing, in which each of the characters stands for a single sound. Egyptian hieroglyphic writing is an example of the former; modern alphabets are examples of the latter.

Maya hieroglyphic writing is ideographic, since its characters represent ideas rather than pictures or sounds. It has been thought by some that there are phonetic elements included in Maya writing.

MAYA WRITING ONE OF THE EARLIEST EXAMPLES OF A GRAPHIC SYSTEM

One of the important facts about the Maya hieroglyphic writing is that, barring such purely pictorial efforts as the paleolithic cave paintings or the American Indian pictographs, it may represent the earliest stage of a formal graphic system that has come down to us. This does not mean that the Maya hieroglyphic writing is the oldest graphic system known. Although the earliest Egyptian and Sumerian inscriptions go back to the fourth millennium before Christ, the earliest known Maya writing was done after the beginning of the Christian Era. However, early Egyptian hieroglyphics had already advanced to a semiphonetic stage. In addition to the many ideographs present, perhaps half the characters are phonetic, mostly syllabic. A similar condition obtains in the earliest cuneiform writing.

The Maya "Rosetta Stone" is the *Relación de las cosas de Yucatán,* written about 1566 by Bishop Diego de Landa. . . .

STORY TOLD BY THE MAYA INSCRIPTIONS

The Maya inscriptions treat primarily of chronology, astronomy, and religious matters. They are not records of personal glorification, like the inscriptions of Egypt, Assyria, and Babylonia. They are so completely impersonal that it is unlikely that the name glyphs of specific men were ever recorded upon the monuments. . . .

MAYA CALENDAR

(Permutations of the Long Count in Maya Calendrics, that is tables of multiples including 52 civil or calendar years—$52 \times 365 = 18{,}980$—which also equals 73 sacred years or almanacs—73×260)

The Maya had two calendars whose dates their priests sought to relate. The first calendar was the sacred year or *tzolkin* of 260 days. Comprised of 13 weeks of 20 days each, it determined the pattern of ceremonial life. The second was the civil calendar or *haab* of 365 days. It contained 19 months—18 months of 20 days each and a final month of 5 days. Relating the dates of one to the other over long periods of time involved determining the least common denominator of 260 and 365. Both numbers divisible by 5; 260 gives a quotient of 52, and 365 gives a quotient of 73, so the least common multiple of 260 and 365 is $5 \times 52 \times 73$, or 18,980. Therefore, Wheel *A* will make 73 revolutions and Wheel *B* will make 52 revolutions before the two wheels

*Source: This selection is taken from Sylvanus G. Morley, *The Ancient Maya* (3rd edition, 1956) and is reprinted by permission of Stanford University Press.

have returned to their original positions, a total of 18,980 elapsed days, or about 52 years.

Once every 52 civil years, then, any given year-bearer coincided with the first day of the year. Thus any Maya who lived more than 52 years began to see New Year's Days of the same name repeat themselves. We do not know the ancient Maya name or hieroglyph for this 52-year period, but modern students of the Maya calendar have called it the Calendar Round.

None of the peoples of Mesoamerica who borrowed their calendars from the Maya made use of time periods longer than this 18,980-day period. The Aztecs, for example, conceived time as an endless succession of these 52-year periods and gave to them the name *xiuhmolpilli*, meaning "year bundle" or complete round of the years. . . .

MAYA ARITHMETIC

In order to escape rapidly mounting calendric chaos, the Maya priests devised a simple numerical system which even today stands as one of the brilliant achievements of the human mind.

Some time during the fourth or third centuries before Christ, the priests devised a system of numeration by position, involving the conception and use of the mathematical quantity of zero, a notable intellectual accomplishment.[1]

The unit of the Maya calendar was the day or *kin*. The second order of units, consisting of 20 kins, was called the *uinal*. In a perfect vigesimal system of numeration, the third term should be 400 (20 × 20 × 1) but at this point the Maya introduced a variation for calendric reckoning. The third order of the Maya system, the *tun*, was composed of 18 (instead of 20) uinals, or 360 (instead of 400) kins. This was a closer approximation to the length of the solar calendar.

Above the third order the unit of progression is uniformly 20, as will be seen from the numerical values of the nine known orders of time periods:

20 *kins*	= 1 *uinal* or 20 days
18 *uinals*	= 1 *tun* or 360 days
20 *tuns*	= 1 *katun* or 7,200 days
20 *katuns*	= 1 *baktun*[2] or 144,000 days
20 *baktuns*	= 1 *pictun* or 2,880,000 days
20 *pictuns*	= 1 *calabtun* or 57,600,000 days
20 *calabtuns*	= 1 *kinchiltun* or 1,152,000,000 days
20 *kinchiltuns*	= 1 *alautun* or 23,040,000,000 days

MAYA GLYPH FORMS

Every Maya hieroglyph occurs in two forms in the inscriptions: (1) the normal form and (2) a head variant, this being the head of a deity, man, animal, bird, serpent, or some mythological creature. Very rarely there is a third form where the glyph is a full figure.

The glyphs for the nine time periods are given in Figure 22, with normal forms at the left and head variants at the right. Head variants have not yet been identified for the last three periods.

MAYA NUMERICAL NOTATIONS

The ancient Maya used two types of notation in writing their numbers: (1) bar-and-dot numerals, and (2) head-variant numerals. In the first notation, the dot • has a numerical value of 1 and the bar ▬ a numerical value of 5, and by varying combinations of these two symbols, the numbers from 1 to 19 were written (Fig. 23). The numbers above 19 were indicated by their positions and will be described later.

Maya bar-and-dot notation was sim-

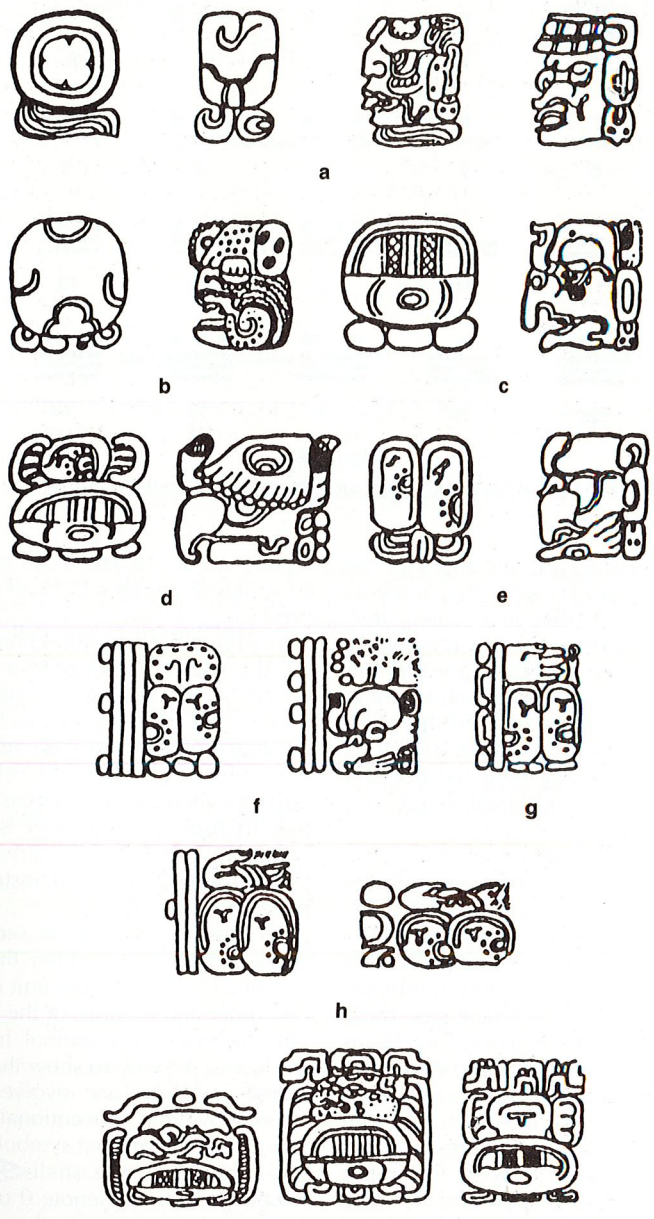

Figure 22

GLYPHS FOR THE NINE KNOWN MAYA TIME-PERIODS: *(A) KIN; (B) UINAL; (C) TUN; (D) KATUN; (E) BAKTUN; (F) PICTUN; (G) CALABTUN; (H) KINCHILTUN; (I) ALAUTUN* OR INITIAL SERIES INTRODUCING-GLYPH.

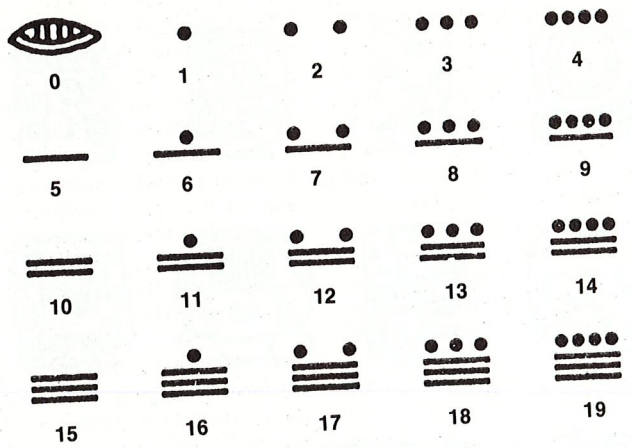

Figure 23

GLYPHS FOR 0 AND THE NUMBERS 1 TO 19 INCLUSIVE.

pler than Roman notation and superior in two respects. To write the numbers from 1 to 19 in Roman notation, it is necessary to employ the symbols I, V, and X, and the processes of addition and subtraction: VI is V plus I, but IV is V minus I. In order to write the same numbers in Maya bar-and-dot, it is necessary to employ only the dot and the bar, and one arithmetical process, that of addition.

The second notation employed in writing Maya numbers used different types of human heads to represent the numbers from 1 to 13, and zero. The Maya head notation is comparable to our Arabic notation, where ten symbols represent zero and the nine digits. These Maya head-variant numerals are heads of the patron deities of the first fourteen numbers (Plate 30a).[3] The head variant for 10 is the death's head, and in forming the head variants for the numbers from 14 to 19 the fleshless lower jaw (Plate 30a) is used to represent the value of 10. For example, if the jaw is applied to the lower part of the head for 6, which is characterized by a pair of crossed sticks in the large-eye socket, the resulting head will be that for 16. It is probable that the heads representing numbers 1 to 13 are those of the *Oxlahuntiku* or Thirteen Gods of the Upper World.

In writing bar-and-dot numbers above 19, the ancient Maya used a positional system of numeration. In our decimal system, the postions to the left of the decimal point increase by tens. In the Maya vigesimal system the values of the positions increase by twenties from bottom to top. An exception is made in counting time, when, as already noted, the third position is 18 instead of 20 times the second.

To illustrate this, let us see how the Maya would have written the number 20, which is 1 complete unit of the second order and no units of the first order. This necessitates a symbol for zero in the lowest position to show that no units of the first order are involved; for this we shall use the conventionalized shell, one of the commonest symbols for zero. Thus, by placing a shell in the lowest position to denote 0 units of the first order and a dot • in the second position to denote 1 unit of the second order, the number 20 can be written (Fig. 24). The manner of writing other numbers is also shown in Figure 24, including two numbers written in the

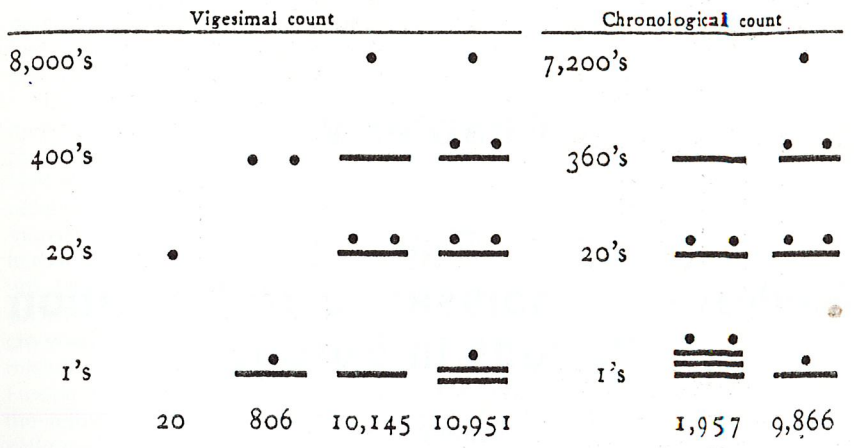

Figure 24

EXAMPLES OF MAYA POSITIONAL MATHEMATICS.

chronological count. The simplicity of Maya addition is also obvious in Figure 24: 10,951, the sum of the numbers in the two preceding columns, is obtained simply by combining the dots and bars of 806 and 10,145 into a new figure.

NOTES

1. **Editor's note.** The invention of place-value notation in Mesoamerica may have been the work of earlier peoples other than the Maya, such as those in Oaxaca, Tabasco, and Vera Cruz.

2. The period of the fifth order, the *baktun*, was originally called the "cycle" by modern investigators. The ancient name for this period, however, was probably *baktun* as given above.

3. The head-variant for the number 11 has not yet been surely identified.