Proposition 47.

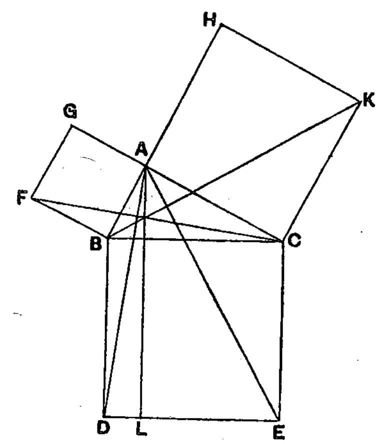
In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

Let *ABC* be a right-angled triangle having the angle *BAC* right;

I say that the square on *BC* is equal to the squares on *BA*, *AC*.

For let there be described on *BC* the square *BDEC*, and on *BA*, *AC* the squares *GB*, *HC*; [I. 46]

through *A* let *AL* be drawn parallel to either *BD* or *CE*, and let *AD*, *FC* be joined.



Then, since each of the angles *BAC*, *BAG* is right, it follows that with a straight line *BA*, and at the point *A* on it, the two straight lines *AC*, *AG* not lying on the same side make the adjacent angles equal to two right angles;

therefore *CA* is in a straight line with *AG*. [I. 14]

For the same reason

BA is also in a straight line with *AH*.

And, since the angle *DBC* is equal to the angle *FBA*: for each is right:

let the angle *ABC* be added to each;

therefore the whole angle *DBA* is equal to the whole angle *FBC*. [C. N. 2]

And, since *DB* is equal to *BC*, and *FB* to *BA*,

the two sides *AB*, *BD* are equal to the two sides *FB*, *BC* respectively,

and the angle *ABD* is equal to the angle *FBC*;

therefore the base *AD* is equal to the base *FC*,

and the triangle *ABD* is equal to the triangle *FBC*. [I. 4]

Now the parallelogram *BL* is double of the triangle *ABD*, for they have the same base *BD* and are in the same parallels *BD*, *AL*. [I. 41]

And the square *GB* is double of the triangle *FBC*, for they again have the same base *FB* and are in the same parallels *FB*, *GC*. [I. 41]

[But the doubles of equals are equal to one another.]

Therefore the parallelogram *BL* is also equal to the square *GB*.

Similarly, if *AE*, *BK* be joined,

the parallelogram *CL* can also be proved equal to the square *HC*;

therefore the whole square *BDEC* is equal to the two squares *GB*, *HC*. [C. N. 2]

And the square *BDEC* is described on *BC*,

and the squares *GB*, *HC* on *BA*, *AC*.

Therefore the square on the side *BC* is equal to the squares on the sides *BA*, *AC*.

Therefore etc. Q. E. D.