lexandria and stayed with me, and
ıat, when I had completed the investi-
ation in eight books, I gave them to
im at once, a little too hastily, because
e was on the point of sailing, and so I
'as not able to correct them, but put
own everything as it occurred to me,
ıtending to make a revision at the end.
ccordingly, as opportunity permits, I
ow publish on each occasion as much
f the work as I have been able to cor-
ect. As certain other persons whom I
ave met have happened to get hold of
ıe first and second books before they
ere corrected, do not be surprised if
ɔu come across them in a different
ırm.

Of the eight books the first four form
ı elementary introduction. The first in-
udes the methods of producing the
ıree sections and the opposite
anches [of the hyperbola] and their
ndamental properties, which are in-
estigated more fully and more gener-
ly than in the works of others. The
cond book includes the properties of
e diameters and the axes of the sec-
ɔns as well the asymptotes, with other
ings generally and necessarily used in
etermining limits of possibility; and
hat I call diameters and axes you will
arn from this book. The third book in-
udes many remarkable theorems use-
l for the syntheses of solid loci and for
etermining limits of possibility; most of
ese theorems, and the most elegant,
e new, and it was their discovery
hich made me realize that Euclid had
ɔt worked out the synthesis of the
cus with respect to three and four
ıes, but only a chance portion of it,
ıd that not successfully; for the synthe-
; could not be completed without the
eorems discovered by me. The fourth
ɔok investigates how many times the
ctions of cones can meet one another
ıd the circumference of a circle; in
ldition it contains other things, none
which have been discussed by previ-
ıs writers, namely, in how many
ıints a section of a cone or a circumference of a circle can meet [the opposite branches of hyperbolas].

The remaining books are thrown in by way of addition: one of them discusses fully *minima* and *maxima*, another deals with equal and similar sections of cones, another with theorems about the determinations of limits, and the last with determinate conic problems. When they are all published it will be possible for anyone who reads them to form his own judgement. Farewell.

### DEFINITIONS

If a straight line be drawn from a point to the circumference of a circle, which is not in the same plane with the point, and be produced in either direction, and if, while the point remains stationary, the straight line be made to move round the circumference of the circle until it returns to the point whence it set out, I call the surface described by the straight line a *conical surface;* it is composed of two surfaces lying vertically opposite to each other, of which each extends to infinity when the straight line which describes them is produced to infinity; I call the fixed point the vertex, and the straight line drawn through this point and the centre of the circle I call the *axis*.

The figure bounded by the circle and the conical surface between the vertex and the circumference of the circle I term a *cone*, and by the *vertex of the cone* I mean the point which is the vertex of the surface, and by the *axis* I mean the straight line drawn from the vertex to the centre of the circle, and by the *base* I mean the circle.

Of cones, I term those *right* which have their axes at right angles to their bases, and *scalene* those which have their axes not at right angles to their bases.

In any plane curve I mean by a *diameter* a straight line drawn from the curve which bisects all straight lines



drawn in the curve parallel to a given straight line, and by the *vertex of the curve* I mean the extremity of the straight line on the curve, and I describe each of the parallels as being drawn *ordinatewise* to the diameter.

Similarly, in a pair of plane curves I mean by a *transverse diameter* a straight line which cuts the two curves and bisects all the straight lines drawn in either curve parallel to a given straight line, and by the *vertices of the curves* I mean the extremities of the diameter on the curves; and by an *erect diameter* I mean a straight line which lies between the two curves and bisects the portions cut off between the curves of all straight lines drawn parallel to a given straight line; and I describe each of the parallels as drawn *ordinate-wise to the diameter*.

By *conjugate diameters* in a curve or pair of curves I mean straight lines of which each, being a diameter, bisects parallels to the other.

By an *axis* of a curve or pair of curves I mean a straight line which, being a diameter of the curve or pair of curves, bisects the parallels at right angles.

By *conjugate axes* in a curve or pair of curves I mean straight lines which, being conjugate diameters, bisect at right angles the parallels to each other.

## 43. From *Conics:* Propositions 7 and 11*

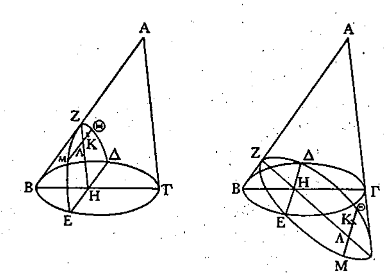
(A Novel Method of Construction of Sections)

APOLLONIUS

### PROPOSITION 7
### [Construction of Sections]

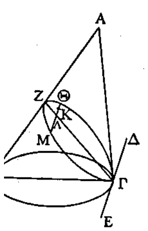
*IF a cone be cut by a plane through the axis, and if it be also cut by another plane cutting the plane containing the base of the cone in a straight line perpendicular to the base of the axial triangle,*[1] *or to the base produced, a section will be made on the surface of the cone by the cutting plane, and straight lines drawn in it parallel to the straight line perpendicular to the base of the axial triangle will meet the common section of the cutting plane and the axial triangle and, if produced to the other part of the section, will be bisected by it; if the cone be right, the straight line in the base will be perpendicular to the common section of the cutting plane and the axial triangle; but if it be scalene, it will not in general be perpendicular, but only when the plane through the axis is perpendicular to the base of the cone.*

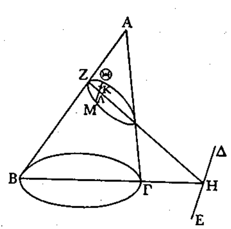
Let there be a cone whose vertex is the point A and whose base is the circle BΓ, and let it be cut by a plane through the axis, and let the section so made be the triangle ABΓ. Now let it be cut by another plane cutting the plane containing the circle BΓ in a straight line ΔE which is either perpendicular to BΓ or to BΓ produced, and let the section made on the surface of the cone be ΔZE[2]; then the common section of the cutting plane and of the triangle ABΓ is ZH. Let any point Θ be taken on ΔZE,



nd through Θ let ΘK be drawn parallel ) ΔE. I say that ΘK intersects ZH and, if roduced to the other part of the section ZE, it will be bisected by the straight ne ZH.

For since the cone, whose vertex is ıe point A and base the circle BΓ, is ut by a plane through the axis and the ection so made is the triangle ABΓ, and ıere has been taken any point Θ on the ırface, not being on a side of the iangle ABΓ, and ΔH is perpendicular ) BΓ, therefore the straight line drawn ırough Θ parallel to ΔH, that is ΘK, ill meet the triangle ABΓ and, if prouced to the other part of the surface, ill be bisected by the triangle. Thereore, since the straight line drawn ırough Θ parallel to ΔE meets the iangle ABΓ and is in the plane containg the section ΔZE, it will fall upon the ommon section of the cutting plane ıd the triangle ABΓ. But the common ction of those planes is ZH; therefore e straight line drawn through Θ parall to ΔE will meet ZH; and if it be proıced to the other part of the section ZE it will be bisected by the straight ıe ZH.




Now the cone is right, or the axial angle ABΓ is perpendicular to the cire BΓ, or neither.

First, let the cone be right; then the angle ABΓ will be perpendicular to e circle BΓ [Eucl. xi. 18]. Then since e plane ABΓ is perpendicular to the ıne BΓ, and ΔE is drawn in one of the ınes perpendicular to their common ction BΓ, therefore ΔE is perpendicu- to the triangle ABΓ [Eucl. xi. Def. 4]; d therefore it is perpendicular to all the straight lines in the triangle ABΓ which meet it [Eucl. xi. Def. 3]. Therefore it is perpendicular to ZH.

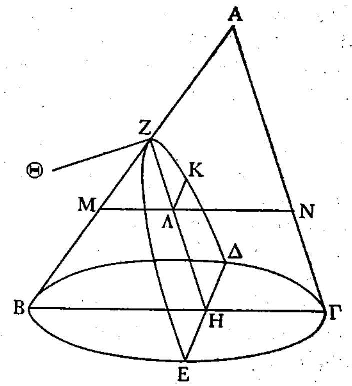
Now let the cone be not right. Then, if the axial triangle is perpendicular to the circle BΓ, we may similarly show that ΔE is perpendicular to ZH. Now let the axial triangle ABΓ be not perpendicular to the circle BΓ. I say that neither is ΔE perpendicular to ZH. For if it is possible, let it be; now it is also perpendicular to BΓ; therefore ΔE is perpendicular to both BΓ, ZH. And therefore it is perpendicular to the plane through BΓ, ZH [Eucl. xi. 4]. But the plane through BΓ, HZ is ABΓ; and therefore ΔE is perpendicular to the triangle ABΓ. Therefore all the planes through it are perpendicular to the triangle ABΓ [Eucl. xi. 18]. But one of the planes through ΔE is the circle BΓ; therefore the circle BΓ is perpendicular to the triangle ABΓ. Therefore the triangle ABΓ is perpendicular to the circle BΓ; which is contrary to hypothesis. Therefore ΔE is not perpendicular to ZH.

**Corollary.** From this it is clear that ZH is a diameter of the section ΔZE, inasmuch as it bisects the straight lines drawn parallel to the given straight line ΔE, and also that parallels can be bisected by the diameter ZH without being perpendicular to it.

## PROPOSITION 11
## [Application of Areas]

*Let a cone be cut by a plane through the axis, and let it be also cut by another plane cutting the base of the cone in a straight line perpendicular to the base of the axial triangle, and further let the diameter of the section be parallel to one side of the axial triangle; then if any straight line be drawn from the section of the cone parallel to the common section of the cutting plane and the base of the cone as far as the diameter of the section, its square will be equal to the rectangle bounded by the intercept made by it on the diameter in the direction of the vertex of the section and a certain other straight line; this straight line will bear the same ratio to the intercept between the angle of the cone and the vertex of the segment as the square on the base of the axial triangle bears to the rectangle bounded by the remaining two sides of the triangle; and let such a section be called a parabola.*






For let there be a cone whose vertex is the point A and whose base is the circle BΓ, and let it be cut by a plane through the axis, and let the section so made be the triangle ABΓ, and let it be cut by another plane cutting the base of the cone in the straight line ΔE perpendicular to BΓ, and let the section so made on the surface of the cone be ΔZE, and let ZH, the diameter of the section, be parallel to AΓ, one side of the axial triangle and from the point Z let ZΘ be drawn perpendicular to ZH, and let $B\Gamma^2 : BA \,.\, A\Gamma = Z\Theta : ZA$, and let any point K be taken at random on the section, and through K let KΛ be drawn parallel to ΔE. I say that $K\Lambda^2 = \Theta Z \,.\, Z\Lambda$.

For let MN be drawn through Λ parallel to BΓ; but KΛ is parallel to ΔE; therefore the plane through KΛ, MN is parallel to the plane through BΓ, ΔE [Eucl. xi. 15], that is to the base of the cone. Therefore the plane through KΛ, MN is a circle, whose diameter is MN [Prop. 4]. And KΛ is perpendicular to MN, since ΔE is perpendicular to BΓ [Eucl. xi. 10];

| | | |
|---|---|---|
| therefore | $M\Lambda \,.\, \Lambda N = K\Lambda^2$. | |
| And since | $B\Gamma^2 : BA \,.\, A\Gamma = \Theta Z : ZA$, | |
| while | $B\Gamma^2 : BA \,.\, A\Gamma = (B\Gamma : \Gamma A)(B\Gamma : BA)$, | |
| therefore | $\Theta Z : ZA = (B\Gamma : \Gamma A)(\Gamma B : BA)$. | |
| But | $B\Gamma : \Gamma A = MN : NA$ | |
| | $= M\Lambda : \Lambda Z$, | [Eucl. vi. 4] |
| and | $B\Gamma : BA = MN : MA$ | |
| | $= \Lambda M : MZ$ | [*ibid.*] |
| | $= N\Lambda : Z A$. | [Eucl. vi. 2] |
| Therefore | $\Theta Z : ZA = (M\Lambda : \Lambda Z)(N\Lambda : ZA)$. | |
| But | $(M\Lambda : \Lambda Z)(\Lambda N : ZA) = M\Lambda \,.\, \Lambda N : \Lambda Z \,.\, ZA$. | |
| Therefore | $\Theta Z : ZA = M\Lambda \,.\, \Lambda N : \Lambda Z \,.\, ZA$. | |
| But | $\Theta Z : ZA = \Theta Z \,.\, Z\Lambda : \Lambda Z \,.\, ZA$, | |
| by taking a common height ZΛ; | | |
| therefore | $M\Lambda \,.\, \Lambda N : \Lambda Z \,.\, ZA = \Theta Z \,.\, Z\Lambda : \Lambda Z \,.\, ZA$. | |
| Therefore | $M\Lambda \,.\, \Lambda N = \Theta Z \,.\, Z\Lambda$ | [Eucl. v. 9] |
| But | $M\Lambda \,.\, \Lambda N = K\Lambda^2$; | |
| and therefore | $K\Lambda^2 = \Theta Z \,.\, Z\Lambda$. | |