4 Archimedes and Apollonius

4.A Archimedes

Though some of his works are lost, we have more of the writings of Archimedes than of any other great mathematician of antiquity. This is a tribute to the high regard in which his work was held, as well as to his productivity. It seems that his works, sent mostly from Syracuse, where he lived, to the mathematical community in Alexandria, continued to be studied sufficiently to ensure their preservation. (For details of their transmission down to us, see 4.B4.) This selection of extracts is representative of his range of mathematical interests—he left few writings about his well-known mechanical inventions—besides containing valuable historical evidence in the prefatory letters. We have followed the ordering of works suggested by Wilbur Knorr (*Archive for History of Exact Sciences*, 19 (1978) pp. 211–290), an ordering that enables the development of Archimedes' mathematical thought and approach to be studied. Archimedes was killed by a Roman soldier in 212 BC.

4.A1 Measurement of a circle

Proposition 1

The area of any circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius, and the other to the circumference, of the circle.

Let *ABCD* be the given circle, *K* the triangle described.
Then, if the circle is not equal to *K*, it must be either greater or less.
I. If possible, let the circle be greater than *K*.
Inscribe a square *ABCD*, bisect the arcs *AB*, *BC*, *CD*, *DA*, then bisect (if necessary) the halves, and so on, until the sides of the inscribed polygon whose angular points are the points of division subtend segments whose sum is less than the excess of the area of the circle over *K*.
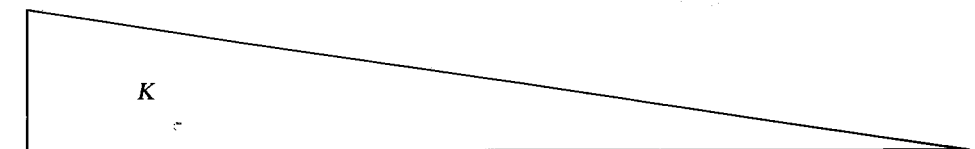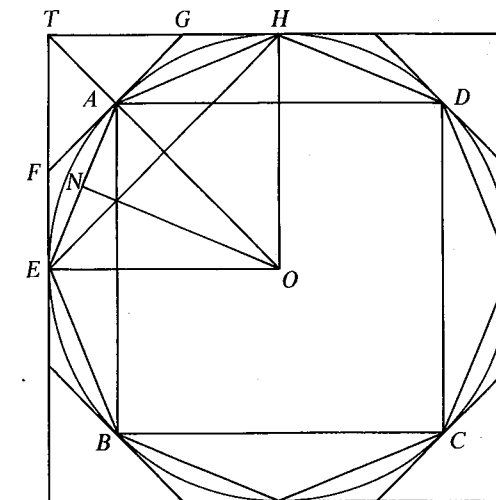
Thus the area of the polygon is greater than *K*.
Let *AE* be any side of it, and *ON* the perpendicular on *AE* from the centre *O*.
Then *ON* is less than the radius of the circle and therefore less than one of the sides about the right angle in *K*. Also the perimeter of the polygon is less than the circumference of the circle, i.e. less than the other side about the right angle in *K*.
Therefore the area of the polygon is less than *K*; which is inconsistent with the hypothesis.
Thus the area of the circle is not greater than *K*.
II. If possible, let the circle be less than *K*.
Circumscribe a square, and let two adjacent sides, touching the circle in *E, H*, meet in *T*. Bisect the arcs between adjacent points of contact and draw the tangents at the points of bisection. Let *A* be the middle point of the arc *EH*, and *FAG* the tangent at *A*.
Then the angle *TAG* is a right angle.
Therefore *TG > GA > GH*.
It follows that the triangle *FTG* is greater than half the area *TEAH*.
Similarly, if the arc *AH* be bisected and the tangent at the point of bisection be drawn, it will cut off from the area *GAH* more than one-half.
Thus, by continuing the process, we shall ultimately arrive at a circumscribed polygon such that the spaces intercepted between it and the circle are together less than the excess of *K* over the area of the circle.
Thus the area of the polygon will be less than *K*.
Now, since the perpendicular from *O* on any side of the polygon is equal to the radius of the circle, while the perimeter of the polygon is greater than the circumference of the circle, it follows that the area of the polygon is greater than the triangle *K*; which is impossible.