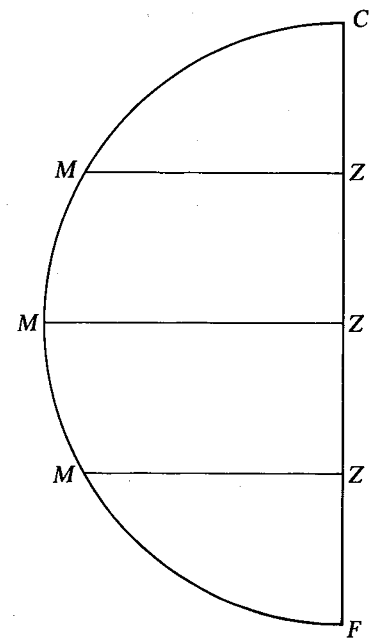
11.E2 Blaise Pascal to Pierre de Carcavy

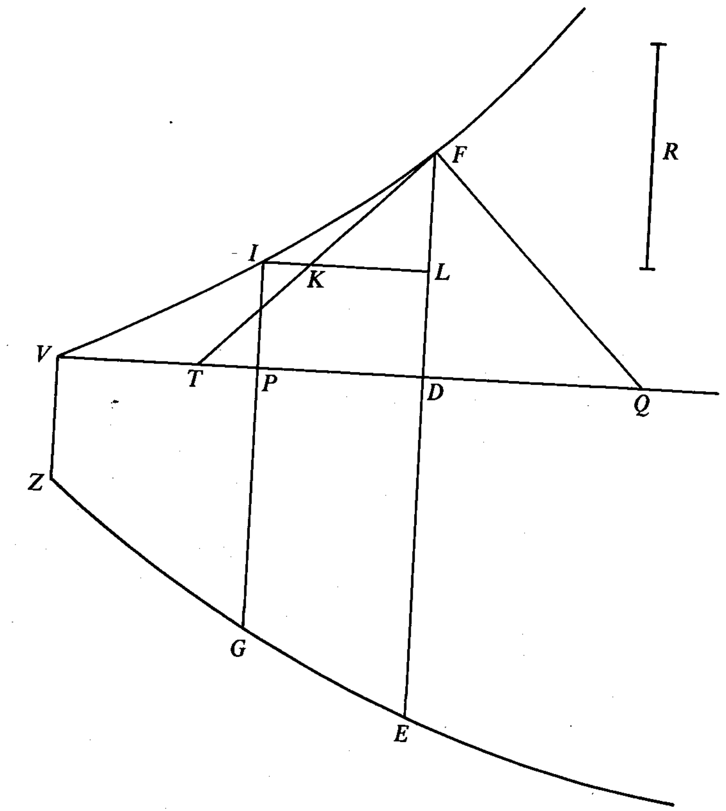
I wanted to write this note to show that everything which is proved by the true rules of indivisibles will also be proved with the rigour and the manner of the ancients, and that therefore the methods differ, the one from the other, only in the way they are expressed: which cannot hurt reasonable people once one has alerted them to what that means.

And that is why I do not find any difficulty in what follows in using the language of indivisibles, the sum of lines or the sum of planes; and thus when for example I consider the diameter of a semicircle divided into an indefinite number of equal parts at the points Z, from which ordinates ZM are taken, I shall find no difficulty in using this expression, the sum of the ordinates, which seems not to be geometric to those who do not understand the doctrine of indivisibles and who imagine that it is to sin against geometry to express a plane by an indefinite number of lines; which only shows their lack of intelligence, for one understands nothing other by that than the sum of an indefinite number of rectangles made on each ordinate with each of the equal portions of the diameter, whose sum is certainly a plane which only differs from the space of the semi-circle by a quantity less than any given quantity.



11.E3 Isaac Barrow on areas and tangents

Let ZGE be any curve of which the axis is VD and let there be perpendicular ordinates to this axis (VZ, PG, DE) continually increasing from the initial ordinate VZ; also let VIF be a line such that, if any straight line EDF is drawn perpendicular to VD, cutting the curves in the points E, F, and VD in D, the rectangle contained by DF and a given length R is equal to the intercepted space $VDEZ$; also let $DE{:}DF = R{:}DT$, and join [T and F]. Then TF will touch the curve VIF. For, if any point I is taken in the line VIF (first on the side of F towards V), and if through it IG is drawn parallel to VZ, and IL is parallel to VD, cutting the given lines as shown in the figure; then $LF{:}LK = DF{:}DT = DE{:}R$, or $R \times LF = LK \times DE$.



But, from the stated nature of the lines DF, LK, we have $R \times LF$ = area $PDEG$: therefore $LK \times DE$ = area $PDEG < DP \times DE$; hence $LK < DP < LI$.

Again, if the point I is taken on the other side of F, and the same construction is made as before, plainly it can be easily shown that $LK > DP > LI$.

From which it is quite clear that the whole of the line TKF lies within or below the curve $VIFI$.

Other things remaining the same, if the ordinates, VZ, PG, DE, continually decrease, the same conclusion is attained by similar argument; only one distinction occurs, namely, in this case, contrary to the other, the curve VIF is concave to the axis VD.

12 Isaac Newton

Sir Isaac Newton (1642–1727) did not dominate even English mathematical and scientific life until the successful publication of his *Philosophiae Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*) in 1687 (see 12.B). Until then the vast bulk of his discoveries lay in his desk drawers, known only in outline to a few friends and colleagues. Afterwards, when he left Cambridge and moved to London, where he was made director of the Royal Mint, he began to publish increasingly, but even so it has remained for modern scholars to print more of Newton's mathematical work than Newton ever did. So the picture of Newton that we have, and the nature of his influence, are necessarily complicated. His first love was for mathematics, and his initial years at Cambridge were spent mastering the literature; works by Oughtred, Wallis, and especially Descartes's *Geometry* and the numerous commentaries on it. But soon he left them behind, and in 1664 began to do his own original research. Our first selections show him at work in this period investigating curves in the Cartesian style, but insisting on the centrality of the problem of tangents (see 12.A1, 12.A3, 12.A5). His use of infinite series lent his work a generality which surpassed Descartes's (see 12.A2, 12.A3), but two other features of his thought are also particularly noteworthy: his emphasis on the tangent as the instantaneous direction of motion along the curve; and his discovery of a pattern in the results which yielded him an algorithm (see 12.A4, 12.A5). Soon he realized that quadrature problems were inverse to tangency problems, and he was then in possession of what can be called the Newtonian calculus.

This calculus makes certain kinds of problems easy which had been difficult, and suggested to Newton that it was now possible to tackle a much harder problem, the inverse tangent problem (raised in 12.A6), which he regarded as a generalization of the finding of areas. His formal rules for 'differentiation' or finding fluxions did not, of course, invert in any simple way, but he found, as his two letters to Leibniz make clear (see 12.C), that his method of infinite series was a great help here too, although it was not the only method.

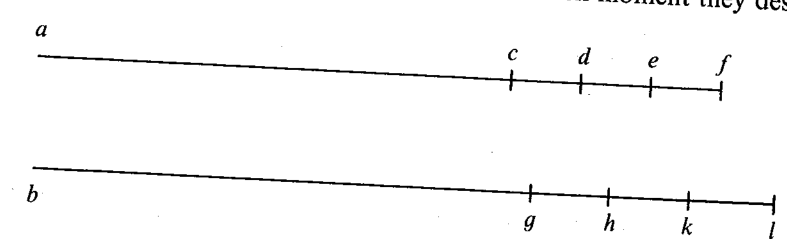
But Newton did not only invent the calculus, and write the *Principia* and his *Opticks*; he was also a first-rate geometer. By this we do not only mean that his geometrical arguments in the *Principia* are skilful and elegant, as they indeed are, but that there and elsewhere he had dramatic new things to say. Another fruit of the 1660s that was not to see the light until his *Opticks* was published in 1704 was his remarkable classification of cubic curves (see 12.D2), which was both a *tour de force* of Cartesian algebraic methods, and the occasion for a lapidary statement of the power of projective geometry. The work was to provoke many eighteenth-century geometers, some of whom we shall look at in 14.D. In the *Principia* itself he not only naturally included many results about conics satisfying various conditions, but also presented his projective transformations of curves (see 12.D4); this, too, gradually brought forth a literature of its own. However, as he grew older, he developed increasingly firm views on the subject of geometry and its superiority over algebra; 12.D3 is just one of many similar passages on this theme.

12.A Newton's Invention of the Calculus

12.A1 Tangents by motion and by the *o*-method

Lemma

If two bodys A, B, move uniformely the $\substack{\text{one}\\\text{other}}$ from $\substack{a\\b}$ to $\substack{c, d, e, f,\\g, h, k, l,}$ &c: in the same time. Then are the lines $\substack{ac,\\bg,}$ & $\substack{cd,\\gh,}$ & $\substack{de,\\hk,}$ & $\substack{ef,\\kl,}$ &c: as their velocitys $\frac{p}{q}$. And though they move not uniformely yet are the infinitely little lines which each moment they describe, as



their velocitys which they have while they describe them. As if the body A with the velocity p describe the infinitely little line $(cd =)p \times o$ in one moment, in that moment the body B with the velocity q will describe the line $(gh =)q \times o$. For $p{:}q{::}po{:}qo$. Soe that if the described lines bee $(ac =)x$, & $(bg =)y$, in one moment, they will bee $(ad =)x + po$, & $(bh =)y + qo$ in the next.

Demonstration

Now if the equation expressing the relation twixt the lines x & y bee $x^3 - abx + a^3 - dyy = 0$. I may substitute $x + po$ & $y + qo$ into the place of x & y; because (by the lemma) they as well as x & y, doe signify the lines described by the