in particular, his own deep study of Diophantus. He acknowledges this in his Preface (8.A5(a)). We also give a glimpse of his profound ideas about complex numbers (8.A5(b)), to which he was led by the awkward fact that when a cubic equation has only one real root, the algebraic method appears to break down. Bombelli saw that the way out was to recognize that the algebra yields the answer as a sum of two complex conjugate numbers and so as a real number after all.

8.A1 Antonio Maria Fior's challenge to Niccolò Tartaglia (1535)

These are the thirty problems proposed by me Antonio Maria Fior to you Master Niccolò Tartaglia.

1 Find me a number such that when its cube root is added to it, the result is six, that is 6. [This is equivalent to the equation $x^3 + x = 6$.]

2 Find me two numbers in double proportion such that when the square of the larger number is multiplied by the smaller, and this product is added to the two original numbers, the result is forty, that is 40. [Equivalent to the equation $(2x)^2 \cdot x + x + 2x = 40$, i.e. $4x^3 + 3x = 40$.]

3 Find me a number such that when it is cubed, and the said number is added to this cube, the result is five. [Equivalent to $x^3 + x = 5$.]

[...]

15 A man sells a sapphire for 500 ducats, making a profit of the cube root of his capital. How much is this profit? [$x^3 + x = 500$.]

[...]

17 There is a tree, 12 *braccia* high, which was broken into two parts at such a point that the height of the part which was left standing was the cube root of the length of the part that was cut away. What was the height of the part that was left standing? [$x^3 + x = 12$.]

[...]

30 There are two bodies of 20 triangular faces [icosahedra] whose corporeal areas added together make 700 *braccia*, and the area of the smaller is the cube root of the larger. What is the smaller area? [$x^3 + x = 700$.]

8.A2 Tartaglia's account of his meeting with Gerolamo Cardano (1539)

CARDANO: I hold it very dear that you have come now, when his Excellency the Signor Marchese has ridden as far as Vigevano, because we will have the opportunity to talk, and to discuss our affairs together until he returns. Certainly you have, alas, been unkind in not wishing to give me the rule that you discovered, on the case of the thing and the cube equal to a number, even after my greatest entreaties for it.

TARTAGLIA: I tell you, I am not so unforthcoming merely on account of the solution, nor of the things discovered through it, but on account of those things which it is possible to discover through the knowledge of it, for it is a key which opens the way

to the ability to investigate boundless other cases. And if it were not that at present I am busy with the translation of Euclid into Italian (and at the moment I have translated as far as his thirteenth book), I would already have found a general rule for many other cases. But as soon as I have completed this work on Euclid that I have already begun, I intend to compose a book on the practice [of arithmetic], and together with it a new algebra, in which I have resolved not only to publish to every man all my discoveries of new cases already mentioned, but many others which I hope to find; and, more, I want to demonstrate the rule that enables one to investigate boundless other cases, which I hope will be a useful and beautiful thing. And this is the reason which makes me refuse them to everyone, because at present I am not working on them (being, as I said, busy with Euclid), and if I teach them to any speculative person (as is your Excellency), he could easily with such clear information find other solutions (it being easy to combine it with the things already discovered), and publish it, as inventor. And to do that would spoil all my plans. Thus this is the principal reason that has made me so unkind to your Excellency, so much more as you are at present having your book printed on a similar subject, and even though you wrote to me that you want to give out these discoveries of mine under my name, acknowledging me as the inventor. Which in effect does not please me on any account, because I want to publish these discoveries of mine in my books, and not in another person's books.

CARDANO: And I also wrote to you that if you did not consent to my publishing them, I would keep them secret.

TARTAGLIA: It is enough that I did not choose to believe that.

CARDANO: I swear to you, by God's holy Gospels, and as a true man of honour, not only never to publish your discoveries, if you teach me them, but I also promise you, and I pledge my faith as a true Christian, to note them down in code, so that after my death no one will be able to understand them. If you want to believe me now, then believe me, if not, leave it be.

TARTAGLIA: If I did not give credit to all your oaths, I would certainly deserve to be judged a faithless man, but since I have decided to ride to Vigevano to call upon his Excellency the Signor Marchese, because it is now three days that I have been here, and I am sorry to have waited for him for so long, when I have returned I promise to demonstrate everything to you.

CARDANO: Since you have decided anyway to ride as far as Vigevano after the Signor Marchese, I want to give you a letter to give to his Excellency, so that he should know who you are. But before you go, I want you to show me the rule for these solutions of yours, as you have promised me.

TARTAGLIA: I am satisfied. But I want you to know, that, to enable me to remember the method in any unforeseen circumstance, I have arranged it as a verse in rhyme, because if I had not taken this precaution, I would frequently have forgotten it, and although my telling it in rhyme is not very concise, it has not bothered me, because it is enough that it serves to bring the rule to mind every time that I recite it. And I want to write down this verse for you in my own hand, so that you can be sure that I am giving you the invention accurately and well.

When the cube and the things together
Are equal to some discrete number,
[To solve $x^3 + cx = d$,]

Find two other numbers differing in this one.
Then you will keep this as a habit
That their product should always be equal
Exactly to the cube of a third of the things.
   [Find u, v such that $u - v = d$ and $uv = (c/3)^3$.]
The remainder then as a general rule
Of their cube roots subtracted
Will be equal to your principal thing.
   [Then $x = \sqrt[3]{u} - \sqrt[3]{v}$.]
In the second of these acts,
When the cube remains alone,
   [In the second case, to solve $x^3 = cx + d$,]
You will observe these other agreements:
You will at once divide the number into two parts
So that the one times the other produces clearly
The cube of a third of the things exactly.
   [Find u, v such that $u + v = d$ and $uv = (c/3)^3$.]
Then of these two parts, as a habitual rule,
You will take the cube roots added together,
And this sum will be your thought.
   [Then $x = \sqrt[3]{u} + \sqrt[3]{v}$.]
The third of these calculations of ours
Is solved with the second if you take good care,
As in their nature they are almost matched.
   [The third case, to solve $x^3 + d = cx$, is similar to the second.]
These things I found, and not with sluggish steps,
In the year one thousand five hundred, four and thirty.
With foundations strong and sturdy
In the city girdled by the sea.

This verse speaks so clearly that, without any other example, I believe that your Excellency will understand everything.

CARDANO: How well I will understand it, and I have almost understood it at the present. Go if you wish, and when you have returned, I will show you then if I have understood it.

TARTAGLIA: Now, remember your Excellency, and just do not forget your faithful promise, because if by unhappy chance it is broken, that is if you publish these solutions, whether in this book that you are having printed at the moment; or even, in another one different from this one, you publish them giving my name and acknowledging me as the real inventor, I promise you and I swear to publish immediately another book which will not be very agreeable for you.

CARDANO: Do not doubt that I will keep my promise. Go, and be sure that you give this my letter to the Signor Marchese on my behalf.

TARTAGLIA: Now please do not forget.

CARDANO: Go, straight away.

TARTAGLIA: By my faith, but for that, I do not want to go to Vigevano. I would much rather turn in the direction of Venice, let the matter go as it will.

8.A3 Tartaglia versus Ludovico Ferrari (1547)

Ferrari to Tartaglia

[…]

15 Find me two numbers such that when they are added together, they make as much as the cube of the lesser added to the product of its triple with the square of the greater; and the cube of the greater added to its triple times the square of the lesser makes 64 more than the sum of these two numbers. [Find a, b such that $a + b = b^3 + 3ba^2$ and $a^3 + 3ab^2 = 64 + a + b$.]

[…]

17 Divide eight into two parts such that their product multiplied by their difference comes to as much as possible, proving everything.

[…]

21 Find me six quantities in continuous proportion starting with one, such that the double of the second with the triple of the third is equal to the root of the sixth. [Find the sequence $1, r, r^2 \ldots r^5$ such that $2r + 3r^2 = \sqrt{r^5}$.]

22 As far as appertains to mathematics, I require the exposition of that place in Plato's *Timaeus*, which, in Latin, begins, '*Fuit autem talis illa partitio*' [He began the division as follows] as far as these words, '*Postquam igitur secundum creatoris*' [Therefore afterwards according to the creator]. [The passage in question is 2.E5(c).]

23 There is a cube such that its sides and its surfaces added together are equal to the proportional quantity between the said cube and one of its faces. What is the size of the cube? [Find x, y such that $12x + 6x^2 = y$, where $\dfrac{x^3}{y} = \dfrac{y}{x^2}$ (i.e. $y^2 = x^5$).]

[…]

27 There is a right-angled triangle, such that when the perpendicular is drawn, one of the sides with the opposite part of the base makes 30, and the other side with the other part makes 28. What is the length of one of the sides? [Find AB, AC or BC such that $AB + DC = 30$ and $AC + BD = 28$.]

[…]

30 Is unity a number or not?
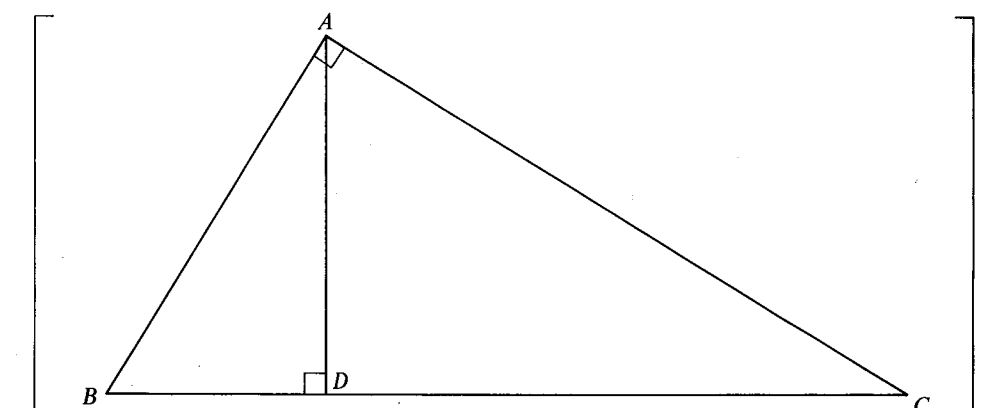
Tartaglia to Ferrari

In your fifteenth problem, you required me to find you two numbers such that, when added together they should make as much as the lesser added to the multiplication of its triple with the square of the greater, and that the cube of the greater together with the multiplication of its triple with the square of the lesser should come to 64 more than the sum of the said two numbers. I reply to you by saying that the greater number, or quantity, will be the cube root of the whole of 4 plus $\sqrt{15\frac{215}{216}}$, plus the cube root of the whole of 4 minus $\sqrt{15\frac{215}{216}}$, plus 2 [i.e. $\sqrt[3]{(4 + \sqrt{15\frac{215}{216}})} + \sqrt[3]{(4 - \sqrt{15\frac{215}{216}})} + 2$]. And the lesser will be the same, minus 2, that is, it will be the cube root of the whole of 4 plus $\sqrt{15\frac{215}{216}}$, plus the cube root of the whole of 4 minus $\sqrt{15\frac{215}{216}}$, minus 2 [$\sqrt[3]{(4 + \sqrt{15\frac{215}{216}})} + \sqrt[3]{(4 - \sqrt{15\frac{215}{216}})} - 2$]. [...]

In your seventeenth problem, you asked me to divide eight into two parts such that their product multiplied by their difference should come to as much as possible. I reply to you that the greater part was 4 plus $\sqrt{5\frac{1}{3}}$ and the lesser was 4 minus $\sqrt{5\frac{1}{3}}$, the product is $10\frac{2}{3}$, which multiplied by the difference, which is $\sqrt{21\frac{1}{3}}$, makes $\sqrt{2423\frac{7}{27}}$, and this is fruit of our tree with which you thought to make war on me, but you failed in that intention. [...]

In your twenty-first problem you required me to find you six quantities in continuous proportion starting with one, and such that the double of the second one with the triple of the third should be equal to the root of the sixth. I reply to you, that the first was (as you required) one, the second of them will be the cube root of the whole of 47 plus $\sqrt{12}$, plus the cube root of the whole of 47 minus $\sqrt{12}$, plus 3 [$\sqrt[3]{(47 + \sqrt{12})} + \sqrt[3]{(47 - \sqrt{12})} + 3$]. The others can be found in the usual way, but since, I accept, there is no skill in that, I omit that labour. [...]

In your twenty-third problem you told me that there is a cube of which the sides and surfaces added together are equal to the quantity in mean proportion between the said cube and one of its faces, and you required of me the size of the cube. I reply to you, that the side of the said cube will be the cube root of the whole of 2664 plus $\sqrt{19008}$, plus the cube root of the whole of 2664 minus $\sqrt{19008}$, plus 12 [$\sqrt[3]{(2664 + \sqrt{19008})} + \sqrt[3]{(2664 - \sqrt{19008})} + 12$]. [...]

To your twenty-seventh problem, where you say, 'There is a right-angled triangle, in which, when the perpendicular is drawn, one of the sides with the opposite part of the base makes 30, and the other [side] with the other [part] makes 28', and you ask me the length of one of the sides. I reply to you, that you have proposed this to me in order that I should elucidate to you what you do not understand, and that it is true, in your *Ars Magna*, Question 14, on page 71 you propose a similar problem, namely, you want one of the sides with the opposite part to make 29, and the other with the other to make 31. In which triangle the sides fixed by you are rational, that is, one would be 20, the other 15, the base 25, the perpendicular 12, the greater part of the base 16, and the lesser 9. And in the end you did not know how to solve such a problem by a general rule. It is a very shameful thing, to put forward such a question in public, and not to know how to solve it by a general rule. I have the same opinion of your Problems 26 and 19, but I reserved to myself my reply to you on them, in front of the referees.

I inform you, moreover, that in my first solutions I interposed one for you which looks credible, but for all that is not true nor properly solved. And I did this for two

reasons. One, to ascertain whether such a question was understood by you or not; the other, to make clear to everyone that in this dispute of ours there is no need for any other referees but ourselves, as I asserted in my first reply. Because, if you had not been ignorant of the solution of that question of yours, you would immediately have noticed its falsity, and you would have made me aware of, or demonstrated, the error in my solution. And I would have assented, to show that in these mathematical disputes, one cannot oppose nor contradict the truth. And then I chose to send it to you to see if you knew the correct solution, but as far as I can see you did not notice that falsely completed solution, which leads me to believe, and to hold for certain, that you are ignorant of the solution to that question.

Ferrari to Tartaglia

My seventeenth problem, if you look at it closely, says, divide eight into two parts such that their product multiplied by their difference comes to as much as possible, proving everything. But you, just like a forger, not only in obscure things but also in open and public things, omit, when reporting my words, the part that matters, and which you did not dare to ignore, namely these two words, 'proving everything', and then having already misused the thing in your way, you simply divide eight into two parts, which I do not wish to tell you, whether they are the correct ones or not. But I will tell you, that, in not proving that which my problem requires, you are still leaving it unsolved, and so you show yourself to be that man of integrity and honour, that I meant in my last *Cartello*. [...]

As for the twenty-second. You at first say that it is not a question for a mathematician. To which I reply, that, if by a mathematician you mean someone like you, that is, someone who spends the whole time on roots, fifth powers, cubes, and other trifles, then you are quite right. And I promise you that if it were up to me to reward you, taking example from the custom of Alexander, I would load you up so much with roots and radishes, that you would never eat anything else in your life. But if by a mathematician you mean a man expert in arithmetic, geometry, astrology, and music, and all the other arts that depend on these ones, as were all the ancient mathematicians; and nowadays there are a few who not only possess the aforementioned arts, but also know how to use them in every other science, as they are required. I tell you that the problem is a mathematical one, and one of the finest that could be posed. Because, whoever understands well the reasoning behind all these numbers, and the reasoning behind these crossings and rotations of lines, and then (which is more important) what follows from these things, will understand the finest passage in the whole of mathematics and philosophy together. I will tell you candidly that this is not a subject for you; and so, interpreting only very clumsily, and some lines on purpose into Italian, you have passed along, leaving the problem wholly unsolved.

8.A4 Gerolamo Cardano

(a) *Cardano's aims in writing the* Ars Magna

This art originated with Mahomet the son of Moses the Arab. Leonardo of Pisa is a trustworthy source for this statement. There remain, moreover, four propositions of