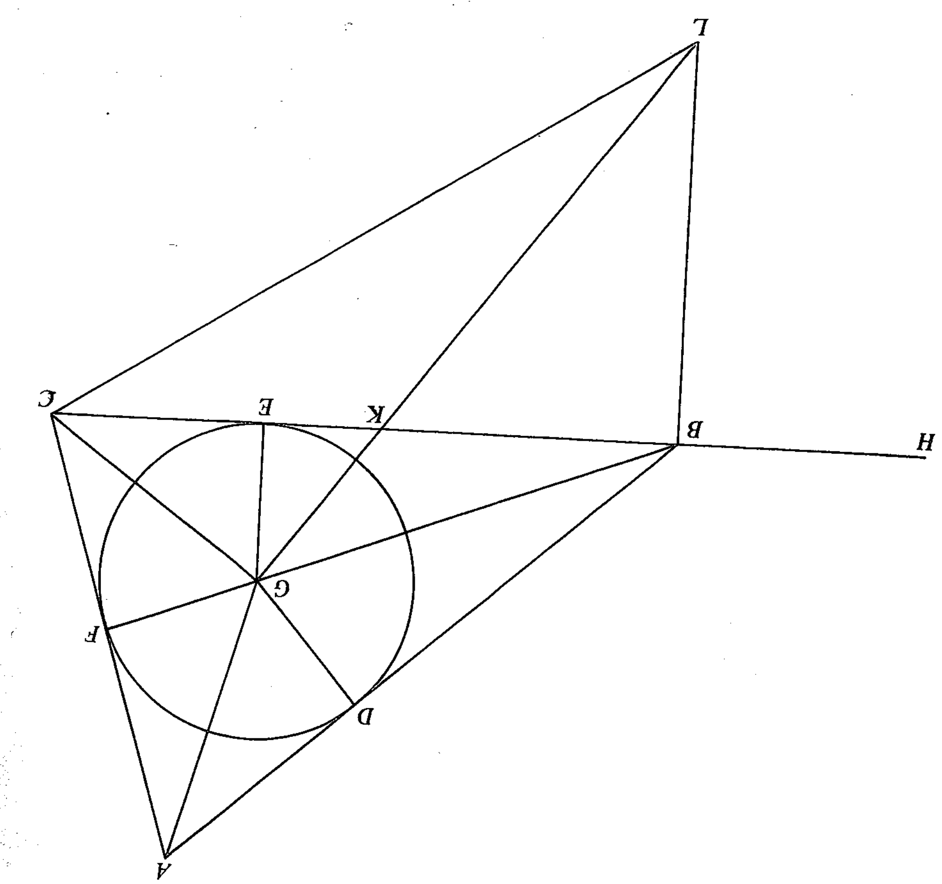
There is a general method for finding, without drawing a perpendicular, the area of any triangle whose three sides are given. For example, let the sides of the triangle be 7, 8 and 9. Add together 7, 8 and 9; the result is 24. Take half of this, which gives 12. Take away 7; the remainder is 5. Again, from 12 take away 8; the remainder is 4. And again 9; the remainder is 3. Multiply 12 by 5; the result is 60. Multiply this by 4; the result is 240. Multiply this by 3; the result is 720. Take the square root of this and it will be the area of the triangle. Since 720 has not a rational square root, we shall make a close approximation to the root in this manner. Since the square nearest to 720 is 729, having a root 27, divide 27 into 720; the result is $26\frac{2}{3}$; add 27; the result is $53\frac{2}{3}$. Take half of this; the result is $26\frac{1}{2}+\frac{1}{3}(=26\frac{5}{6})$. Therefore the square root of 720 will be very nearly $26\frac{5}{6}$. For $26\frac{5}{6}$ multiplied by itself gives $720\frac{1}{36}$; so that the difference is $\frac{1}{36}$. If we wish to make the difference less than $\frac{1}{36}$, instead of 729 we shall take the number now found, $720\frac{1}{36}$, and by the same method we shall find an approximation differing by much less than $\frac{1}{36}$.

The geometrical proof of this is as follows: *In a triangle whose sides are given to find the area.* Now it is possible to find the area of the triangle by drawing one perpendicular and calculating its magnitude, but let it be required to calculate the area without the perpendicular.

Let ABC be the given triangle, and let each of AB, BC, CA be given; to find the area. Let the circle DEF be inscribed in the triangle with centre G [Euclid's *Elements* IV. 9], and let AG, BG, CG, DG, EG, FG be joined. Then [Euclid I. 41] $BC\cdot EG = 2\cdot\text{triangle } BGC$, $CA\cdot FG = 2\cdot\text{triangle } AGC$, $AB\cdot DG = 2\cdot\text{triangle } ABG$. Therefore the rectangle contained by the perimeter of the triangle ABC and EG, that is the radius of the circle DEF, is double of the triangle ABC. Let CB be produced and let BH be placed equal to AD; then CBH is half of the perimeter of the triangle ABC because $AD = AF$, $DB = BE$, $FC = CE$ [by Euclid III. 17]. Therefore $CH\cdot EG = \text{triangle } ABC$. But $CH\cdot EG = \sqrt{CH^2\cdot EG^2}$; therefore $(\text{triangle } ABC)^2 = HC^2\cdot EG^2$.

Let GL be drawn perpendicular to CG and BL perpendicular to CB, and let CL be joined. Then since each of the angles CGL, CBL is right, a circle can be described about the quadrilateral $CGBL$ [Euclid III. 31]; therefore the angles CGB, CLB are together equal to two right angles [Euclid III. 22]. But the angles CGB, AGD are together equal to two right angles because the angles at G are bisected by AG, BG, CG and the angles CGB, AGD together with AGC, DGB are equal to four right angles; therefore, the angle AGD is equal to the angle CLB. But the right angle ADG is equal to the right angle CBL; therefore the triangle AGD is similar to the triangle CBL.

Therefore $BC:BL = AD:DG = BH:EG$, and so [Euclid V. 16] $BC:BH = BL:EG = BK:KE$, because BL is parallel to GE; hence [Euclid V. 18]



$CH:BH = BE:EK$. Therefore $CH^2:CH\cdot HB = BE\cdot EC:CE\cdot EK = BE\cdot EC:EG^2$, for in a right-angled triangle EG has been drawn from the right angle perpendicular to the base; therefore $CH^2\cdot EG^2$, whose square root is the area of the triangle ABC, is equal to $(CH\cdot HB)(CE\cdot EB)$. And each of CH, HB, BE, CE is given; for CH is half of the perimeter of the triangle ABC, while BH is the excess of half the perimeter over CB, BE is the excess of half the perimeter over AC, and CE is the excess of half the perimeter over AB, inasmuch as $EC = CF$, $BH = AD = AF$. Therefore the area of the triangle ABC is given.