What Is a Limit?

*Leibniz (1684):* If any continuous transition is proposed terminating in a certain limit, then it is possible to form a general reasoning, which covers also the final limit.

*Newton (1687):* The ultimate ratio of evanescent quantities ... [are] limits towards which the ratios of quantities decreasing without limit do always converge; and to which they approach nearer than by any given difference, but never go beyond, nor in effect attain to, till the quantities are diminished in infinitum.

*Maclaurin (1742):* The ratio of $2x + e$ to $a$ continually decreases while $e$ decreases and is always greater than the ratio of $2x$ to $a$ while $e$ is any real increment, but it is manifest that it continually approaches to the ratio of $2x$ to $a$ as its limit.

*D'Alembert (1754):* This ratio $[a : 2y + z]$ is always smaller than $a : 2y$, but the smaller $z$ is, the greater the ratio will be and, since one may choose $z$ as small as one pleases, the ratio $a : 2y + z$ can be brought as close to the ratio $a : 2y$ as we like. Consequently, $a : 2y$ is the limit of the ratio $a : 2y + z$.

*Lacroix (1806):* The limit of the ratio $(u_1 - u)/h$ ... is the value towards which this ratio tends in proportion as the quantity $h$ diminishes, and to which it may approach as near as we choose to make it.

*Cauchy (1821):* If the successive values attributed to the same variable approach indefinitely a fixed value, such that they finally differ from it by as little as one wishes, this latter is called the limit of all the others.