1 ORESME. THE LATITUDE OF FORMS

We have taken from medieval authors only those selections (I.1, II.1) that establish a direct link with the Arabic world. A special Source Book will be necessary to deal with the medieval scholastic contribution to science. However, we make an exception for the work of Oresme, since there are indications that this forms a direct link with Renaissance science.

The study of the Aristotelian categories of quality and quantity focused attention on the difference between the *intensio* and *remissio* of a quality and the increase and decrease of a quantity. A quantity, as a solid, increases by adding another body to it, but a quality, as wisdom, is intensified in quite another way, and so is heat, considered a quality before, in the seventeenth century, thermometry was invented. Another quality was motion, and change in velocity was widely discussed, notably in the thirteenth and fourteenth centuries. This gave rise to the application of quantitative ideas to qualities.

One form in which this application occurred was that of *calculatio*, which applied the Euclidean theory of proportions to theological concepts, and to qualities including motion. There was indeed in the work of some of the *Calculatores* (such as Thomas Bradwardine, archbishop of Canterbury, *c.* 1290–1349) a beginning of kinematics. There was also some work on infinite series in connection with Zeno's paradoxes, for instance, by Richard Suiseth, of Merton College, Oxford, *c.* 1345, called *the* Calculator; see C. B. Boyer, *The history of the calculus* (Dover, New York, 1959), 74–79.

Another form was the geometrical representation of the variability of the intensity of a quality, and with this is connected the name of Nicole Oresme (*c.* 1323–1382), M.A. of the University of Paris (1349), who was dean of Rouen Cathedral (1361–1377) and afterward Bishop of Lisieux. He represented, as one of the first to do so, such a variable value, and especially that of a velocity, for any point of a body or for any instant of time by a line segment plotted in a given direction, and thus drawing the first graph.

We find this idea in several works written by him or ascribed to him, of which the *Tractatus de latitudinibus formarum*, probably written by a pupil, was published in Padua (1482). There were later editions, which show that Oresme's ideas may have had circulation among Renaissance mathematicians. He uses the concept of *uniformity* and *difformity* (combined in *uniformiter difformis*, uniformly difform), when the change of intensity upon displacement along the base, the *longitudo*, is proportional to the amount of this displacement. To every point of the *longitudo* corresponds a line segment called *latitudo*, indicating the intensity. The plotted latitudes form a plane surface, and so we obtain the notion of a "rectangular" heat, when the intensity is uniform, or a "trapezoidal" velocity when the velocity increases uniformly. The end points of the latitudes form the graph, the *linea summitatis* (summit line).

We take our selection from a manuscript by Oresme recently published, *Quaestiones super geometriam Euclides*, ed. H. L. L. Busard (Brill, Leiden, 1961), with an English translation, from which we take the vital Question 10. It is preceded in Questions 1–9 by a number of different topics current in fourteenth-century scholastic mathematics, such as the discussion of the existence of an infinite circle and the commensurability or incommensurability of the diagonal and the side of a square.

In Question 10 and in the seven following questions Oresme expounds his theory of … preceding question. The first question he

ferent qualities can be represented. He concludes the question with a spirited and highly personal defense, in which he appeals, among others, to Aristotle.

Oresme considers a place or body (*subjectum*) in which he takes a straight line segment, the *longitudo*, and at each point of it, perpendicular to the segment in a plane through it, a line segment representing the value of a certain intensity.

Further information on the *Calculatores* and Oresme can be found in E. J. Dijksterhuis, *The mechanization of the world picture* (Clarendon Press, Oxford, 1961), 185–200. On the *Quaestiones* see also J. Murdock, "Oresme's Commentary to Euclid," *Scripta Mathematica 27* (1964), 67–91, and Selection V.9. For other mathematical works by Oresme see his *De proportionibus proportionum and Ad pauca respicientes*, ed. and trans. E. Grant (University of Wisconsin Press, Madison, 1966), and Marshall Clagett, *The science of mechanics in the Middle Ages* (University of Wisconsin Press, Madison, 1959), who brings to our attention that a certain Giovanni di Casali may have used a graph in 1346, hence before Oresme, whose work dates from between 1348 and 1362 (pp. 332–333, 414).

(*a*) The altitude of a surface is judged by the line drawn perpendicular to the base, as might appear from a figure.

(*b*) A surface is called uniformly or equally high if all the lines by which the altitude is judged are equal; a surface is called difformly high if these lines are unequal and extend up to a line not parallel to the base (the summit line).

(*c*) The altitude is called uniformly difform if every three or more equidistant altitudes exceed one another in an arithmetical proportion [Fig. 1], i.e., the

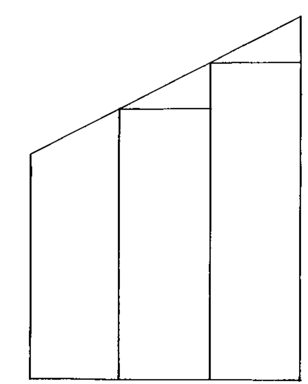

Fig. 1

first extends as much above the second as the second above the third, from which it appears that the summit line is a straight line which is not parallel to the base.

(*d*) The altitude is called difformly difform if the altitudes do not exceed one another in this way. In that case their summit line is not straight and the difformity in altitude varies with the variation of this summit line.

Furthermore:

(*a*) Of a quality two things are represented, *viz.* the *intensio per gradum* and the *extensio per subjectum*, and consequently such a quality is imagined to have two dimensions.

H. L. L. Busard remarks that this means that the extensity of an extended quality of any kind can be designated by a line or a plane (called the longitude) described in the subject. The intensity of the quality from point to point in the subject has to be represented by lines (called latitudes) erected perpendicular to the longitude of the same quality. The latitude thus acts as a variable ordinate in a system of coordinates, while the longitude is not to be identified with the variable abscissa; there is only one longitude with an infinite number of latitudes.[1]

For this reason it is sometimes said that a quality has a latitude and a longitude instead of an intensity and an extensity.

(*b*) A quality may be imagined as belonging to a point or an indivisible subject, such as the soul, but also to a line and even to a surface and a body.

Conclusion 1: The quality of a point or an indivisible subject can be represented by a line, because it has only one dimension, *viz.* intensity. From this it follows that such a quality, such as knowledge or virtue, cannot be called uniform or difform, just as a line is not called uniform or difform. It also follows that properly speaking one cannot refer to the latitude of knowledge and virtue, because no longitude can be associated with it, whereas every latitude presupposes a longitude.

Conclusion 2: The quality of a line can be represented by a surface, of which the longitude is the rectilineal extensity of the subject and the latitude the intensity, which is represented by perpendiculars erected on the subject-line.

Conclusion 3: Similarly the quality of a surface can be represented by a body of which the length and the breadth form the extensity of the surface and the depth is the intensity of the quality. For the same reason the quality of a whole body might be represented by a body of which the length and the breadth would be the extensity of the whole body and the depth the intensity of the quality.

However, someone might doubt: If the quality of a line is here represented by a surface and the quality of a surface by a body having three dimensions, the quality of a body will no doubt be represented by something having four dimensions in a different kind of quantity.

I say that it is not necessary to give a fourth dimension. In fact, if one … *fluens* causes a line, a line a surface, a surface a body, it

is not necessary for a *corpus fluens* to cause a fourth kind of quantity but only a body, and because of this Aristotle, in *De Caelo* I,[2] says that according to this method of representation no passage from a body to a different kind of quantity is possible. In the case under consideration one should reason in the same way.

It is therefore necessary to speak of the quality of a line, and analogously it is considered what has to be said of the quality of a surface or a body.

Conclusion 4: A uniform linear quality can be represented by a rectangle that is uniformly high, in such a way that the base represents the extensity and the summit line is parallel to the base.

A uniformly difform quality can be represented by a surface that is uniformly difformly high, in such a way that the summit line is not parallel to the base. This can be proved: the intensities of the points of the quality are proportional to the altitudes of the perpendiculars erected in the corresponding points of the base.

A quality can be uniformly difform in two ways, just as a surface can be uniformly difformly high in two ways:

(*a*) Such a quality may terminate at zero degree and is then represented by a surface which is uniformly difformly high down to zero, i.e., by a right-angled triangle;

(*b*) Such a quality may terminate at both ends at some degree and is then represented by a quadrilateral, the summit line of which is a straight line not parallel to the base, i.e., by a right-angled trapezium.

Conclusion 5: By means of the above it can be proved that a uniformly difform quality is equal to the medium degree [Fig. 2], i.e., just as great as a uniform quality would be at the degree of the middle point, and this can be proved in the same way as for a surface.[3]

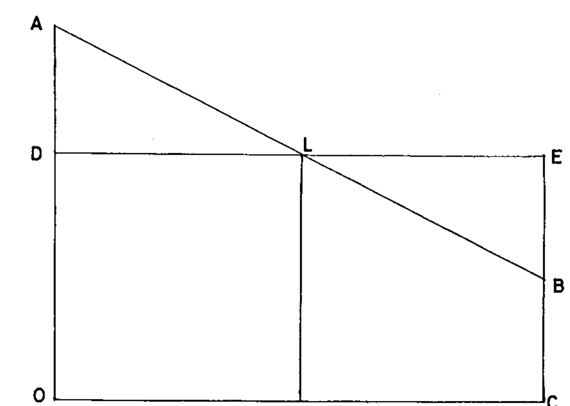

Fig. 2

[2] This rejection of a fourth dimension is based on Aristotle's *De Caelo*, I, 1; 268a31–268b2. It remains the prevailing attitude in Renaissance days even where, as in Cardan or Viète, quadratic equations are related to planes and cubic equations to solids (see Selections II.3, 5). Despite an occasional remark by Pascal, Wallis, and Lagrange, only the nineteenth century took the geometry of four dimensions seriously. See Selections III.3, note 3, and IV.12.

[3] Conclusion 5 is often …

Last conclusion: A difformly difform quality is represented by a surface of which the line representing the subject is the base, while the summit line is a nonstraight line, not parallel to the base. Such a difformity may be imagined in an infinite number of different ways, for the summit line may vary in a great many ways.

However, someone might say: It is not necessary to represent a quality in this way. I say that the representation is good, as also appears in Aristotle,[4] for he represents time by a line. In the same way in *Perspectiva* the *virtus activa* is represented by a triangle.[5] Moreover according to this representation one can understand more easily what is said about uniformly difform qualities, and consequently the representation is good.

This means that, since qualities are represented by surfaces, the equality of two surfaces may also be transferred to the qualities which they represent. In this case, therefore, one has to prove that surface $OCBA$ = surface $OCED$, and from this equality it then follows that the uniformly difform quality that is represented by $OCBA$ is equal to the uniform quality that is represented by $OCED$.

2 REGIOMONTANUS. TRIGONOMETRY

Trigonometry was developed into a independent branch of mathematics by Islamic writers, notably by Nasir ed-dīn at-Tūsī (or Nasir Eddin, 1201–1274). The first publication in Latin Europe to achieve the same goal was Regiomontanus' *De triangulis omnimodis* (On triangles of all kinds; Nuremberg, 1533).

[4] Aristotle, *Physica*, IV, 11; 220a4–20. In lines 219b1–2 Aristotle defines time as "numerus motus secundum prius et posterius." Here he tries to explain that the "now-moment," on the one hand, divides time into two parts (past–future), but, on the other hand, makes it continuous. He compares time to a line on which a point makes a division but also constitutes continuity on the line.

[5] The *virtus activa* is the light diffused from the source of light (*lumen*). Later, in Question 17, Oresme concludes: "Such a force or such a light extends uniformly difformly, or in other words: it is a uniformly difform quality. This appears plausible because—since the force does not extend uniformly—it seems to diminish as the distance increases; this diminution has to take place proportionally, i.e., uniformly difformly" [Fig. 3]. The *Perspectiva* mentioned is the one written by Witelo (Vitellio), a Polish mathematician of the thirteenth

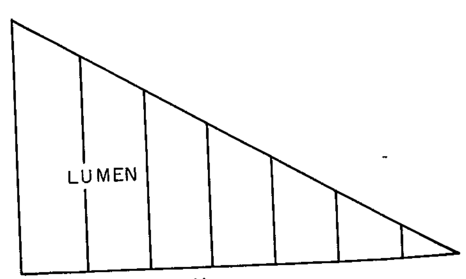

Fig. 3

XX.

In omni triangulo rectangulo, si super uertice acuti anguli, secundũ quantitatem lateris maximi circulum descripseris, erit latus ipsum acutum, subtendens angulum, sinus rectus conterminalis sibi arcus dictũ angulum respicientis: lateri autem tertio sinus complementi arcus dicti æqualis habebitur.

Sit triangulus rectanglus a b c, angulum c rectum habens, & a acutum, super cuius uertice a secundum quãtitatem lateris maximi a b, maximo scilicet angulo oppositi describatur circulus b e d, cuius circũferentiæ occurrat latus a c quoad satis est prolongatũ in e puncto. Dico quòd latus b c angulo b a c oppositum est sinus arcus b e dictum angulum subtendentis. Latus autem tertium, scilicet a c, æquale est sinui recto complementi arcus b e. Extendatur enim latus b c occurrendo circumferentiæ circuli in puncto d. à punctis autem a quidem centro circuli exeat semidiameter a k æquedistans lateri b c. & à puncto b corda b h æquedistans lateri a c. secabunt autem se necessario duæ lineæ b h & a k, angulis a b h & b a k acutis existentibus, quod fiat in puncto g. Quia itaq; semidiameter a e cordam b d secat orthogonaliter propter angulum a c b rectum, secabit & ipsam per æqualia ex tertia tertij elementorũ. & arcum b d per æqualia ex 29. eiusdem. quemadmodum igitur tota linea b d per diffinitionem corda est arcus b d, ita medietas eius, linea scilicet b c est sinus dimidij arcus b e respicientis angulũ b a e siue b a c. quod asseruit prima pars theorematis nostri. Secundam deinceps partem ueram cõfiteberis, si prius per 34. primi angulum a g b rectum esse didiceris. semidiameter enim a K, & cordam b h, & arcũ eius ex supra memoratis medijs per æqua scindet. quare per diffinitionem linea recta b g sinus erit arcus b K. Est autem linea b g æqualis la

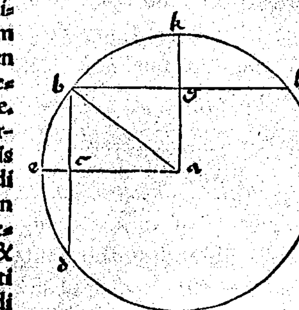

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Fig. 1