sixty-four: we call them the 364 *commencements*, described thus 364⓪, and so of all other like.

THE THIRD DEFINITION

*And each tenth part of the unity of the COMMENCEMENT we call the PRIME, whose sign is thus ①, and each tenth part of the unity of the prime we call the SECOND, whose sign is ②, and so of the other: each tenth part of the unity of the precedent sign, always in order one further.*

*Explication.* As 3① 7② 5③ 9④, that is to say: 3 *primes*, 7 *seconds*, 5 *thirds*, 9 *fourths*, and so proceeding infinitely, but to speak of their value, you may note that according to this definition the said numbers are $\frac{3}{10}$, $\frac{7}{100}$, $\frac{5}{1000}$, $\frac{9}{10000}$, together $\frac{3759}{10000}$, and likewise 8⓪ 9① 3② 7③ are worth 8, $\frac{9}{10}$, $\frac{3}{100}$, $\frac{7}{1000}$, together $8\frac{937}{1000}$, and so of other like. Also you may understand that in this *dime* we use no fractions, and that the multitude of signs, except ⓪, never exceed 9, as for example not 7① 12②, but in their place 8① 2②, for they value as much.

THE FOURTH DEFINITION

*The numbers of the second and third definitions beforegoing are generally called DIME NUMBERS.*

The End of the Definitions

THE SECOND PART OF THE DIME.
Of the Operation or Practice.

THE FIRST PROPOSITION: OF ADDITION

*Dime numbers being given, how to add them to find their sum.*

*The Explication Propounded:* There are 3 orders of dime numbers given, of which the first 27⓪, 8①, 4②, 7③, the second 37⓪, 6①, 7②, 5③, the third 875⓪, 7①, 8②, 2③.

*The Explication Required:* We must find their total sum.

*Construction.* The numbers given must be placed in order as here adjoining, adding them in the vulgar manner of adding of whole numbers in this manner. The sum (by the first problem of our French Arithmetic[2]) is 941304, which are (that which the signs above the numbers do show) 941⓪ 3① 0② 4③. I say they are the sum required.

| | | | | | |
|---|---|---|---|---|---|
| | | ⓪ | ① | ② | ③ |
| | 2 | 7 | 8 | 4 | 7 |
| | 3 | 7 | 6 | 7 | 5 |
| 8 | 7 | 5 | 7 | 8 | 2 |
| 9 | 4 | 1 | 3 | 0 | 4 |

*Demonstration.* The 27⓪ 8① 4② 7③ given make by the 3rd definition before 27, $\frac{8}{10}$, $\frac{4}{100}$, $\frac{7}{1000}$, together $27\frac{847}{1000}$ and by the same reason the 37⓪ 6① 7② 5③ shall make $37\frac{675}{1000}$ and the 875⓪ 7① 8② 2③ will make $875\frac{782}{1000}$, which three numbers make by common addition of vulgar arithmetic $941\frac{304}{1000}$. But so much is the sum 941⓪ 3① 0② 4③; therefore it is the true sum to be demonstrated. Conclusion: Then dime numbers being given to be added, we have found their sum, which is the thing required.

[2] *L'Arithmétique de Simon Stevin de Bruges* (Leyden, 1585); see Stevin, *The principal works* (Swets-Zeitlinger, Amsterdam), vol. IIB (1958). Problem I (p. 81) is: "Given two arithmetical integer numbers. Find their sum."



*Note* that if in the number given there want some signs of their natural order, the place of the defectant shall be filled. As for example, let the numbers given be 8⓪ 5① 6② and 5⓪ 7②, in which the latter wanted the sign of ①; in the place thereof shall 0① be put. Take then for that latter number given 5⓪ 0① 7②, adding them in this sort.

| | | | |
|---|---|---|---|
| | ⓪ | ① | ② |
| | 8 | 5 | 6 |
| | 5 | 0 | 7 |
| 1 | 3 | 6 | 3 |

This advertisement shall also serve in the three following propositions, wherein the order of the defailing figures must be supplied, as was done in the former example.

THE SECOND PROPOSITION: OF SUBTRACTION

*A dime number being given to subtract, another less dime number given: out of the same to find their rest.*

*Explication Propounded:* Be the numbers given 237⓪ 5① 7② 8③ and 59⓪ 7① 3② 9③.

*The Explication Required:* To find their rest.

*Construction.* The numbers given shall be placed in this sort, subtracting according to vulgar manner of subtraction of whole numbers, thus.

| | | | | | |
|---|---|---|---|---|---|
| | | ⓪ | ① | ② | ③ |
| 2 | 3 | 7 | 5 | 7 | 8 |
| | 5 | 9 | 7 | 3 | 9 |
| 1 | 7 | 7 | 8 | 3 | 9 |

The rest is 177839, which values as the signs over them do denote 177⓪ 8① 3② 9③, I affirm the same to be the rest required.

*Demonstration.* The 237⓪ 5① 7② 8③ make (by the third definition of this Dime) $237\frac{5}{10}$, $\frac{7}{100}$, $\frac{8}{1000}$, together $237\frac{578}{1000}$, and by the same reason the 59⓪ 7① 3② 9③ value $59\frac{739}{1000}$, which subtracted from $237\frac{578}{1000}$, there rests $177\frac{839}{1000}$, but so much doth 177⓪ 8① 3② 9③ value; that is then the true rest which should be made manifest.

*Conclusion.* A dime being given, to subtract it out of another dime number, and to know the rest, which we have found.

THE THIRD PROPOSITION: OF MULTIPLICATION

*A dime number being given to be multiplied, and a multiplicator given: to find their product.*

*The Explication Propounded:* Be the number to be multiplied 32⓪ 5① 7②, and the multiplicator 89⓪ 4① 6②.

*The Explication Required:* To find the product.

*Construction.* The given numbers are to be placed as here is shown, multiplying according to the vulgar manner of multiplcation by whole numbers, in this manner, giving the product 29137122. Now to know how much they value, join the two last signs together as the one ② and the other ② also, which together make ④, and say that the last sign of the product shall be ④, which being known, all the rest are also known by their continued order. So that the product required is 2913⓪ 7① 1② 2③ 2④.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | ⓪ | ① | ② |
| | | | | 3 | 2 | 5 | 7 |
| | | | | 8 | 9 | 4 | 6 |
| | | | 1 | 9 | 5 | 4 | 2 |
| | | 1 | 3 | 0 | 2 | 8 | |
| | 2 | 9 | 3 | 1 | 3 | | |
| 2 | 6 | 0 | 5 | 6 | | | |
| 2 | 9 | 1 | 3 | 7 | 1 | 2 | 2 |
| | | | ⓪ | ① | ② | ③ | ④ |

*Demonstration.* The number given to be multiplied, 32⓪ 5① 7② (as appears by the third definition of this Dime), $32\frac{5}{10}$, $\frac{7}{100}$, together $32\frac{57}{100}$; and by the same reason the multiplicator 89⓪ 4① 6② value $89\frac{46}{100}$ by the same, the said $32\frac{57}{100}$ multiplied gives the product $2913\frac{7122}{10000}$. But it also values 2913⓪ 7① 1② 2③ 2④.

It is then the true product, which we were to demonstrate. But to show why ② multiplied by ② gives the product ④, which is the sum of their numbers, also why ④ by ⑤ produces ⑨, and why ⓪ by ③ produces ③, etc., let us take $\frac{2}{10}$ and $\frac{3}{100}$, which (by the third definition of this Dime) are 2① 3②, their product is $\frac{6}{1000}$, which value by the said third definition 6③; multiplying then ① by ②, the product is ③, namely a sign compounded of the sum of the numbers of the signs given.

*Conclusion.* A dime number to multiply and to be multiplied being given, we have found the product, as we ought.

*Note: If the latter sign of the number to be multiplied be unequal to the latter sign of the multiplicator, as, for example, the one 3④ 7⑤ 8⑥, the other 5① 4②, they shall be handled as aforesaid, and the disposition thereof shall be thus.*

```
      ④ ⑤ ⑥
      3  7  8
         5  4 ②
   ----------
   1  5  1  2
1  8  9  0
-------------
2  0  4  1  2
④ ⑤ ⑥ ⑦ ⑧
```

THE FOURTH PROPOSITION: OF DIVISION

*A dime number for the dividend and divisor being given: to find the quotient.*

*Explication Proposed:* Let the number for the dividend be 3⓪ 4① 4② 3③ 5④ 2⑤ and the divisor 9① 6②.

*Explication Required:* To find their quotient.

*Construction.* The numbers given divided (omitting the signs) according to the vulgar manner of dividing of whole numbers, gives the quotient 3587; now to know what they value, the latter sign of the divisor ② must be subtracted from the latter sign of the dividend, which is ⑤, rests ③ for the latter are also manifest by their continued order, thus 3⓪ 5① 8② 7③ are the quotient required.

```
  1
  18
 5164
 7617     ⓪ ① ② ③
344352   (3  5  8  7
 96666
  999
```

*Demonstration.* The number dividend given 3⓪ 4① 4② 3③ 5④ 2⑤ makes (by the third definition of this Dime) 3, $\frac{4}{10}$, $\frac{4}{100}$, $\frac{3}{1000}$, $\frac{5}{10000}$, $\frac{2}{100000}$, together $3\frac{44352}{100000}$, and by the same reason the divisor 9① 6② values $\frac{96}{100}$, by which $3\frac{44352}{100000}$ being divided, gives the quotient $3\frac{587}{1000}$; but the said quotient values 3⓪ 5① 8② 7③, therefore it is the true quotient to be demonstrated.

*Conclusion.* A dime number being given for the dividend and divisor, we have found the quotient required.

*Note: If the divisor's signs be higher than the signs of the dividend, there may be as many such ciphers* 0 *joined to the dividend as you will, or as many as shall be necessary: as for example,* 7② *are to be divided by* 4⑤, *I place after the* 7 *certain* ⓪, *thus* 7000, *dividing them as afore said, and in this sort it gives for the quotient* 1750⓪.

```
3 2
7 0 0 0  (1 7 5 0 ⓪
4 4 4 4
```



It happens also sometimes that the quotient cannot be expressed by whole numbers, as 4① divided by 3② in this sort, whereby appears that there will infinitely come 3's, and in such a case you may come so near as the thing requires, omitting the remainder. It is true, that 13⓪ 3① $3\frac{1}{3}$②, or 13⓪ 3① 3② $3\frac{1}{3}$③ etc. shall be the perfect quotient required. But our invention in this Dime is to work all by whole numbers. For seeing that in any affairs men reckon not of the thousandth part of a mite, es, grain, etc., as the like is also used of the principal geometers and astronomers in computations of great consequence, as Ptolemy and Johannes Montaregio,[3] have not described their tables of arcs, chords, or sines in extreme perfection (as possibly they might have done by multinomial numbers), because that imperfection (considering the scope and end of those tables) is more convenient than such perfection.

```
1 1 1 (1              ⓪ ① ②
4 0 0 0  0  0  0  (1  3  3  3
3 3 3 3
```

*Note* 2. The extraction of all kinds of roots may also be made by these dime numbers; as, for example, to extract the square root of 5② 2③ 9④, which is performed in the vulgar manner of extraction in this sort, and the root shall be 2① 3②, for the moiety or half of the latter sign of the numbers given is always the latter sign of the root; wherefore, if the latter sign given were of a number impair, the sign of the next following shall be added, and then it shall be a number pair; and then extract the root as before. Likewise in the extraction of the cubic root, the third part of the latter sign given shall be always the sign of the root; and so of all other kinds of roots.

```
1
5 2 9
2   3
-----
  4
```

THE END OF THE DIME

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After this follows an Appendix in which different applications of the decimal method of counting to surveying, cloth measuring, wine gauging, and other trades and professions are described. The decimal division of weights and measures was not systematically introduced until the French Revolution. As to its introduction (and nonintroduction) into the United States, see C. D. Hellman, "Jefferson's efforts towards the decimalization of U.S. weights and measures," *Isis 16* (1931), 266–314.

## 4 NAPIER. LOGARITHMS

John Napier (or Neper, 1550–1617), a Scottish baron, computed a table of what he called logarithms, using the correspondence between an arithmetic and a geometric progression. He published his invention first in the *Mirifici logarithmorum canonis descriptio* (Edinburgh,

[3] Johannes Montaregio (1436–1476) is best known under his latinized name, Johannes Regiomontanus. This craftsman, humanist, astronomer, and mathematician of Nuremberg influenced the development of trigonometry by means of his widely used book *De triangulis omnimodis* (written c. 1464, printed in Nuremberg in 1533). The sines, for Regiomontanus as well as for Stevin, were half chords; see our note to the following text on Napier and Selection III.2.