INTERVENTION CENTRAL

RTI Toolkit: A Practical Guide for Schools

RTI: Interventions for Elementary Math Difficulties

Jim Wright, Presenter

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Jim Wright
,364 Long Road
.Tully, NY 13159
Email: jim@jimwrightonline.com
Download workshop resources at:
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Profile of Students With Significant Math Difficulties (Rourke, 1993)	*NOTES*
Spatial organization. The student commits errors such as misaligning numbers in columns in a multiplication problem or confusing directionality in a subtraction problem (and subtracting the original number—minuend—from the figure to be subtracted (subtrahend).	
Visual detail. The student misreads a mathematical sign or leaves out a decimal or dollar sign in the answer.	
Procedural errors. The student skips or adds a step in a computation sequence. Or the student misapplies a learned rule from one arithmetic procedure when completing another, different arithmetic procedure.	
Inability to 'shift psychological set'. The student does not shift from one operation type (e.g., addition) to another (e.g., multiplication) when warranted.	
Graphomotor. The student's poor handwriting can cause him or her to misread handwritten numbers, leading to errors in computation.	
Memory. The student fails to remember a specific math fact needed to solve a problem. (The student may KNOW the math fact but not be able to recall it at 'point of performance'.)	
Judgment and reasoning. The student comes up with solutions to problems that are clearly unreasonable. However, the student is not able adequately to evaluate those responses to gauge whether they actually make sense in context.	

Reference

Rourke, B. P. (1993). Arithmetic disabilities, specific & otherwise: A neuropsychological perspective. *Journal of Learning Disabilities, 26,* 214-226.

How Do We Reach Low-Performing Math Students?: Instructional Recommendations *Important elements of math instruction for low-performing students* (Baker, Gersten, & Lee, 2002; p. 51):	*IDEAS FOR IMPLEMENTATION*
"Providing teachers and students with data on student performance"	
"Using peers as tutors or instructional guides"	
"Providing clear, specific feedback to parents on their children's mathematics success"	
"Using principles of explicit instruction in teaching math concepts and procedures."	

Reference

Baker, S., Gersten, R., & Lee, D. (2002).A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal, 103*(1), 51-73.

School-Wide Strategies for Managing... MATHEMATICS

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Mathematics instruction is a lengthy, incremental process that spans all grade levels. As children begin formal schooling in kindergarten, they develop 'number sense', an intuitive understanding of foundation number concepts and relationships among numbers. A central part of number sense is the student's ability to internalize the number line as a precursor to performing mental arithmetic. As students progress through elementary school, they must next master common math operations (addition, subtraction, multiplication, and division) and develop fluency in basic arithmetic combinations ('math facts'). In later grades, students transition to applied, or 'word', problems that relate math operations and concepts to real-world situations. Successful completion of applied problems requires that the student understand specialized math vocabulary, identify the relevant math operations needed to solve the problem while ignoring any unnecessary information also appearing in that written problem, translate the word problem from text format into a numeric equation containing digits and math symbols, and then successfully solve. It is no surprise, then, that there are a number of potential blockers to student success with applied problems, including limited reading decoding and comprehension skills, failure to acquire fluency with arithmetic combinations (math facts), and lack of proficiency with math operations. Deciding what specific math interventions might be appropriate for any student must therefore be a highly individualized process, one that is highly dependent on the student's developmental level and current math skills, the requirements of the school district's math curriculum, and the degree to which the student possesses or lacks the necessary auxiliary skills (e.g., math vocabulary, reading comprehension) for success in math. Here are some wide-ranging classroom (Tier I RTI) ideas for math interventions that extend from the primary through secondary grades.

Applied Problems: Encourage Students to Draw to Clarify Understanding *(Van Essen & Hamaker, 1990; Van Garderen, 2006)*. Making a drawing of an applied, or 'word', problem is one easy heuristic tool that students can use to help them to find the solution. An additional benefit of the drawing strategy is that it can reveal to the teacher any student misunderstandings about how to set up or solve the word problem. To introduce students to the drawing strategy, the teacher hands out a worksheet containing at least six word problems. The teacher explains to students that making a picture of a word problem sometimes makes that problem clearer and easier to solve. The teacher and students then independently create drawings of each of the problems on the worksheet. Next, the students show their drawings for each problem, explaining each drawing and how it relates to the word problem. The teacher also participates, explaining his or her drawings to the class or group. Then students are directed independently to make drawings as an intermediate problem-solving step when they are faced with challenging word problems. NOTE: This strategy appears to be more effective when used in later, rather than earlier, elementary grades.

Applied Problems: Improving Performance Through a 4-Step Problem-Solving Approach *(Pólya, 1957; Williams, 2003)*. Students can consistently perform better on applied math problems if they follow an efficient 4-step plan of understanding the problem, devising a plan, carrying out the plan, and looking back. (1) UNDERSTAND THE PROBLEM. To fully grasp the problem, the student may restate the problem in his or her own words, note key information, and identify missing information. (2) DEVISE A PLAN. In mapping out a strategy to solve the problem, the student may make a table, draw a diagram, or translate the verbal problem into an equation. (3) CARRY OUT THE PLAN. The student implements the steps in the plan, showing work and checking work for each step. (4) LOOK BACK. The student checks the results. If the answer is written as an equation, the student puts the results in words and checks whether the answer addresses the question posed in the original word problem.

Math Computation: Boost Fluency Through Explicit Time-Drills *(Rhymer, Skinner, Jackson, McNeill, Smith & Jackson, 2002; Skinner, Pappas & Davis, 2005; Woodward, 2006)*. Explicit time-drills are a method to boost students' rate of responding on math-fact worksheets. The teacher hands out the worksheet. Students are told that they will have 3 minutes to work on problems on the sheet. The teacher starts the stop watch and tells the students to start work. At the end of the first minute in the 3-minute span, the teacher 'calls time', stops the stopwatch, and tells the students to underline the last number written and to put their pencils in the air. Then students are told to resume work and the teacher restarts the stopwatch. This process is repeated at the end of minutes 2 and 3. At the conclusion of the 3 minutes, the teacher collects the student worksheets. TIPS: Explicit time-drills work best on 'simple' math facts requiring few computation steps. They are less effective on more complex math facts. Also, a less intrusive and more flexible version of this intervention is to use time-prompts while students are working independently on math facts to speed their rate of responding. For example, at the end of every minute of seatwork, the teacher can call the time and have students draw a line under the item that they are working on when that minute expires.

Math Computation: Motivate With 'Errorless Learning' Worksheets *(Caron, 2007)*. Reluctant students can be motivated to practice math number problems to build computational fluency when given worksheets that include an answer key (number problems with correct answers) displayed at the top of the page. In this version of an 'errorless learning' approach, the student is directed to complete math facts as quickly as possible. If the student comes to a number problem that he or she cannot solve, the student is encouraged to locate the problem and its correct answer in the key at the top of the page and write it in. Such speed drills build computational fluency while promoting students' ability to visualize and to use a mental number line. TIP: Consider turning this activity into a 'speed drill'. The student is given a kitchen timer and instructed to set the timer for a predetermined span of time (e.g., 2 minutes) for each drill. The student completes as many problems as possible before the timer rings. The student then graphs the number of problems correctly computed each day on a time-series graph, attempting to better his or her previous score.

Math Computation: Two Ideas to Jump-Start Active Academic Responding *(Skinner, Pappas & Davis, 2005)*. Research shows that when teachers use specific techniques to motivate their classes to engage in higher rates of active and accurate academic responding, student learning rates are likely to go up. Here are two ideas to accomplish increased academic responding on math tasks. First, break longer assignments into shorter assignments with performance feedback given after each shorter 'chunk' (e.g., break a 20-minute math computation worksheet task into 3 seven-minute assignments). Breaking longer assignments into briefer segments also allows the teacher to praise struggling students more frequently for work completion and effort, providing an additional 'natural' reinforcer. Second, allow students to respond to easier practice items orally rather than in written form to speed up the rate of correct responses.

Math Homework: Motivate Students Through Reinforcers, Interesting Assignments, Homework Planners, and Self-Monitoring *(Bryan & Sullivan-Burstein, 1998)*. Improve students' rate of homework completion and quality by using reinforcers, motivating 'real-life' assignments, a homework planner, and student self-monitoring. (1) Reinforcers: Allow students to earn a small reward (e.g., additional free time) when they turn in all homework assignments for the week. (2) 'Real-life' Assignments: Make homework meaningful by linking concepts being taught to students' lives. In a math lesson on estimating area, for example, give students the homework task of calculating the area of their bedroom and estimating the amount of paint needed to cover the walls. (3) Homework Planner: Teach students to use a homework planner to write down assignments, organize any materials (e.g., worksheets) needed for homework, transport completed homework safely back to school, and provide space for parents and teachers to communicate about homework via written school-home notes. (4) Student Self-Monitoring: Direct students to chart their homework completion each week. Have students plot the number of assignments turned in on-time in green, assignments not turned in at all in red, and assignments turned in late in yellow.

Math Instruction: Consolidate Student Learning During Lecture Through the Peer-Guided Pause *(Hawkins, & Brady, 1994)*. During large-group math lectures, teachers can help students to retain more instructional content by incorporating brief Peer Guided Pause sessions into lectures. Students are trained to work in pairs. At one or more appropriate review points in a lecture period, the instructor directs students to pair up to work together for 4 minutes. During each Peer Guided Pause, students are given a worksheet that contains one or more correctly completed word or number problems illustrating the math concept(s) covered in the lecture. The sheet also contains several additional, similar problems that pairs of students work cooperatively to complete, along with an answer key. Student pairs are reminded to (a) monitor their understanding of the lesson concepts; (b) review the correctly math model problem; (c) work cooperatively on the additional problems, and (d) check their answers. The teacher can direct student pairs to write their names on the practice sheets and collect them as a convenient way to monitor student understanding.

Math Instruction: Increase Student Engagement and Improve Group Behaviors With Response Cards *(Armendariz & Umbreit, 1999; Lambert, Cartledge, Heward & Lo, 2006)*. Response cards can increase student active engagement in group math activities while reducing disruptive behavior. In the group-response technique, all students in the classroom are supplied with an erasable tablet ('response card'), such as a chalk slate or laminated white board with erasable marker. The teacher instructs at a brisk pace. The instructor first poses a question to the class. Students are given sufficient wait time for each to write a response on his or her response card. The teacher then directs students to present their cards. If most or all of the class has the correct answer, the teacher praises the group. If more than one quarter of the students records an incorrect answer on their cards, however, the teacher uses guided questions and demonstration to steer students to the correct answer.

Math Instruction: Maintain a Supportive Atmosphere for Classroom "Math Talk" *(Cooke & Adams, 1998)*. Teachers can promote greater student 'risk-taking' in mathematics learning when they cultivate a positive classroom atmosphere for math discussions while preventing peers from putting each other down. The teacher models behavioral expectations for open, interactive discussions, praises students for their class participation and creative attempts at problem-solving, and regularly points out that incorrect answers and misunderstandings should be celebrated—as they often lead to breakthroughs in learning. The teacher uses open-ended comments (e.g., "What led you to that answer?") as tools to draw out students and encourage them to explore and apply math concepts in group discussion. Students are also encouraged in a supportive manner to evaluate each other's reasoning. However, the teacher intervenes immediately to prevent negative student comments or 'put-downs' about peers. As with any problem classroom behavior, a first offense requires that the student meet privately with the instructor to discuss teacher expectations for positive classroom behavior. If the student continues to put down peers, the teacher imposes appropriate disciplinary consequences.

Math Instruction: Support Students Through a Wrap-Around Instruction Plan *(Montague, 1997; Montague, Warger & Morgan, 2000)*. When teachers instruct students in more complex math cognitive strategies, they must support struggling learners with a 'wrap-around' instructional plan. That plan incorporates several elements: (a) Assessment of the student's problem-solving skills. The instructor first verifies that the student has the necessary academic competencies to learn higher-level math content, including reading and writing skills, knowledge of basic math operations, and grasp of required math vocabulary. (b) Explicit instruction. The teacher presents new math content in structured, highly organized lessons. The instructor also uses teaching tools such as Guided Practice (moving students from known material to new concepts through a thoughtful series of teacher questions) and 'overlearning' (teaching and practicing a skill with the class to the point at which students develop automatic recall and control of it). (c) Process modeling. The teacher adopts a 'think aloud' approach, or process modeling, to verbally reveal his or her cognitive process to the class while using a cognitive strategy to solve a math problem. In turn, students are encouraged to think aloud when applying the same strategy—first as part of a whole-class or cooperative learning group, then independently. The teacher observes students during process modeling to verify that they are correctly applying the cognitive strategy. (d) Performance feedback. Students get regular performance feedback about their level of mastery in learning the cognitive strategy. That feedback can take many forms, including curriculum-based measurement, timely corrective feedback, specific praise and encouragement, grades, and brief teacher conferences. (e) Review of mastered skills or material. Once the student has mastered a cognitive strategy, the teacher structures future class lessons or independent work to give the student periodic opportunities to use and maintain the strategy. The teacher also provides occasional brief 'booster sessions', reteaching steps of the cognitive strategy to improve student retention.

Math Instruction: Unlock the Thoughts of Reluctant Students Through Class Journaling *(Baxter, Woodward & Olson, 2005)*. Students can effectively clarify their knowledge of math concepts and problem-solving strategies through regular use of class 'math journals'. Journaling is a valuable channel of communication about math issues for students who are unsure of their skills and reluctant to contribute orally in class. At the start of the year, the teacher introduces the journaling assignment, telling students that they will be asked to write and submit responses at least weekly to teacher-posed questions. At first, the teacher presents 'safe' questions that tap into the students' opinions and attitudes about mathematics (e.g., 'How important do you think it is nowadays for cashiers in fast-food restaurants to be able to calculate in their head the amount of change to give a customer?"). As students become comfortable with the journaling activity, the teacher starts to pose questions about the students' own mathematical thinking relating to specific assignments. Students are encouraged to use numerals, mathematical symbols, and diagrams in their journal entries to enhance their explanations. The teacher provides brief written comments on individual student entries, as well as periodic oral feedback and encouragement to the entire class on the general quality and content of class journal responses. Regular math journaling can prod students to move beyond simple 'rote' mastery of the steps for completing various math problems toward a deeper grasp of the math concepts that underlie and explain a particular problem-solving approach. Teachers will find that journal entries are a concrete method for monitoring student understanding of more abstract math concepts. To promote the quality of journal entries, the teacher might also assign them an effort grade that will be calculated into quarterly math report card grades.

Math Problem-Solving: Help Students Avoid Errors With the 'Individualized Self-Correction Checklist' *(Zrebiec Uberti, Mastropieri & Scruggs, 2004)*. Students can improve their accuracy on particular types of word and number problems by using an 'individualized self-instruction checklist' that reminds them to pay attention to their own specific error patterns. To create such a checklist, the teacher meets with the student. Together they analyze common error patterns that the student tends to commit on a particular problem type (e.g., 'On addition problems that require carrying, I don't always remember to carry the number from the previously added column.'). For each type of error identified, the student and teacher together describe the appropriate step to take to prevent the error from occurring (e.g., 'When adding each column, make sure to carry numbers when needed.'). These self-check items are compiled into a single checklist. Students are then encouraged to use their individualized self-instruction checklist whenever they work independently on their number or word problems. As older students become proficient in creating and using these individualized error checklists, they can begin to analyze their own math errors and to make their checklists independently whenever they encounter new problem types.

Math Review: Balance Massed & Distributed Practice *(Carnine, 1997)*. Teachers can best promote students acquisition and fluency in a newly taught math skill by transitioning from massed to distributed practice. When students have just acquired a math skill but are not yet fluent in its use, they need lots of opportunities to try out the skill under teacher supervision—a technique sometimes referred to as 'massed practice'. Once students have developed facility and independence with that new math skill, it is essential that they then be required periodically to use the skill in order to embed and retain it—a strategy also known as 'distributed practice'. Teachers can program distributed practice of a math skill such as reducing fractions to least common denominators into instruction either by (a) regularly requiring the student to complete short assignments in which they practice that skill in isolation (e.g., completing drill sheets with fractions to be reduced), or (b) teaching a more advanced algorithm or problem-solving approach that incorporates--and therefore requires repeated use of--the previously learned math skill (e.g., requiring students to reduce fractions to least-common denominators as a necessary first step to adding the fractions together and converting the resulting improper fraction to a mixed number).

Math Review: Teach Effective Test-Preparation Strategies *(Hong, Sas, & Sas, 2006)*. A comparison of the methods that high and low-achieving math students typically use to prepare for tests suggests that struggling math students need to be taught (1) specific test-review strategies and (2) time-management and self-advocacy skills. Among review-related strategies, deficient test-takers benefit from explicit instruction in how to take adequate in-class notes; to adopt a systematic method to review material for tests (e.g., looking over their notes each night, rereading relevant portions of the math text, reviewing handouts from the teacher, etc.), and to give themselves additional practice in solving problems (e.g., by attempting all homework items, tackling additional problems from the text book, and solving problems included in teacher handouts). Deficient test-takers also require pointers in how to allocate and manage their study time wisely, to structure their study environment to increase concentration and reduce distractions, as well as to develop 'self-advocacy' skills such as seeking additional help from teachers when needed. Teachers can efficiently teach effective test-preparation methods as a several-session whole-group instructional module.

Math Vocabulary: Preteach, Model, and Use Standard Math Terms *(Chard, D., n.d.)*. Three strategies can help students to learn essential math vocabulary: preteaching key vocabulary items, modeling those vocabulary words, and using only universally accepted math terms in instruction. (1) Preteach key math vocabulary. Math vocabulary provides students with the language tools to grasp abstract mathematical concepts and to explain their own reasoning. Therefore, do not wait to teach that vocabulary only at 'point of use'. Instead, preview relevant math vocabulary as a regular a part of the 'background' information that students receive in preparation to learn new math concepts or operations. (2) Model the relevant vocabulary when new concepts are taught. Strengthen students' grasp of new vocabulary by reviewing a number of math problems with the class, each time consistently and explicitly modeling the use of appropriate vocabulary to describe the concepts being taught. Then have students engage in cooperative learning or individual practice activities in which they too must successfully use the new vocabulary—while the teacher provides targeted support to students as needed. (3) Ensure that students learn standard, widely accepted labels for common math terms and operations and that they use them consistently to describe their math problem-solving efforts.

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Building Number Sense Through a Counting Board Game

DESCRIPTION: The student plays a number-based board game to build skills related to 'number sense', including number identification, counting, estimation skills, and ability to visualize and access specific number values using an internal number-line (Siegler, 2009).

MATERIALS:

- *Great Number Line Race!* Form (attached)
- Spinner divided into two equal regions marked "1" and "2" respectively. (NOTE: If a spinner is not available, the interventionist can purchase a small blank wooden block from a crafts store and mark three of the sides of the block with the number "1" and three sides with the number "2".)

INTERVENTION STEPS: A counting-board game session lasts 12 to 15 minutes, with each game within the session lasting 2-4 minutes. Here are the steps:

1. *Introduce the Rules of the Game.* If the student is unfamiliar with the counting board game, interventionist trains the student to play it.

   The student is told that he or she will attempt to beat another player (either another student or the interventionist). The student is then given a penny or other small object to serve as a game piece. The student is told that players takes turns spinning the spinner (or, alternatively, tossing the block) to learn how many spaces they can move on *the Great Number Line Race!* board. Each player then advances the game piece, moving it forward through the numbered boxes of the game-board to match the number "1" or "2" selected in the spin or block toss.

   When advancing the game piece, the player must call out the number of each numbered box as he or she passes over it. For example, if the player has a game piece on box 7 and spins a "2", that player advances the game piece two spaces, while calling out "8" and "9" (the names of the numbered boxes that the game piece moves across during that turn).

   The player who reaches the "10" box first is the winner.

2. *Record Game Outcomes.* At the conclusion of each game, the interventionist records the winner using the form found on the *Great Number Line Race!* form. The session continues with additional games being played for a total of 12-15 minutes.

3. *Continue the Intervention Up to an Hour of Cumulative Play.* The counting-board game continues until the student has accrued a total of at least one hour of play across multiple days. (The amount of cumulative play can be calculated by adding up the daily time spent in the game as recorded on the *Great Number Line Race!* form.)

Reference

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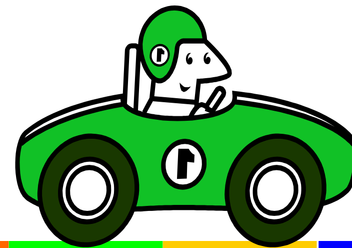
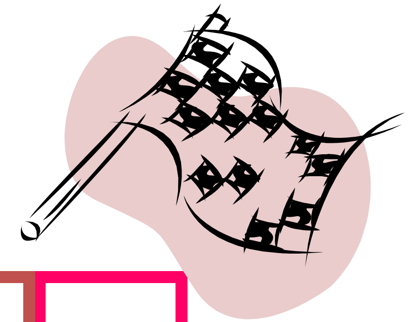
The Great Number-Line Race!





Start	1	2	3	4	5	6	7	8	9	10

Date: ______________________ Start Time: _____ : _____ End Time: _____ : _____

Directions: Mark the winner for each game with an 'X' in the table below.

Players	Game 1	Game 2	Game 3	Game 4	Game 5	Game 6	Game 7
1: _________							
2: _________							

Source: Siegler, R. S. (2009). Improving the numerical understanding of children from low-income families. *Child Development Perspectives*, *3*(2), 1

Reducing the Student's Memorization Load: Math 'Shortcuts'

Students who struggle with math computation may benefit from being taught math 'shortcuts' that lighten the cognitive load (Gersten, Jordan & Flojo, 2005). Here are suggested shortcuts for the basic math operations:

Addition (Miller, Strawser & Mercer, 1996)

- ❑ The order of the numbers in an addition problem does not affect the answer.
- ❑ When zero is added to the original number, the answer is the original number.
- ❑ When 1 is added to the original number, the answer is the next larger number.

- ❑ ADDITION: Strategic Count-Up Strategy (Fuchs et al., 2009):
    1. The student is given a copy of the number-line.
    2. When presented with a two-addend addition problem, the student is taught to start with the larger of the two addends and to 'count up' by the amount of the smaller addend to arrive at the answer to the problem.

Subtraction (Miller, Strawser & Mercer, 1996)

- ❑ When zero is subtracted from the original number, the answer is the original number.
- ❑ When 1 is subtracted from the original number, the answer is the next smaller number.
- ❑ When the original number has the same number subtracted from it, the answer is zero.

- ❑ SUBTRACTION: Strategic Count-Up Strategy (Fuchs et al., 2009):
    1. The student is given a copy of the number-line.
    2. The student is taught to refer to the first number appearing in the subtraction problem (the minuend) as 'the number you start with' and to refer to the number appearing after the minus (subtrahend) as 'the minus number'.
    3. The student is directed to start at the minus number on the number-line and to count up to the starting number while keeping a running tally of numbers counted up on his or her fingers.
    4. The final tally of digits separating the minus number and starting number is the answer to the subtraction problem.

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Multiplication (Miller, Strawser & Mercer, 1996)

- ❑ When a number is multiplied by zero, the answer is zero.
- ❑ When a number is multiplied by 1, the answer is the original number.
- ❑ When a number is multiplied by 2, the answer is equal to the number being added to itself.
- ❑ The order of the numbers in a multiplication problem does not affect the answer.

MULTIPLICATION: Strategic Count-By Strategy (Cullinan, Lloyd & Epstein, 1981)

1. The student looks at the two terms of the multiplication problem. The student picks one of the terms as a number that he or she can count by (the 'count by' number).
2. The student takes the remaining term from the multiplication problem (the 'count times' number) and makes a corresponding number of tally marks to match it.
3. The student starts counting using the 'count by' number. While counting, the student touches each of the tally marks matching the 'count times' number.
4. The student stops counting when he or she has reached the final tally-mark. The student writes down the last number said as the answer to the multiplication problem.

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Division (Miller, Strawser & Mercer, 1996)

- ❑ When zero is divided by any number, the answer is zero.
- ❑ When a number is divided by 1, the answer is the original number.
- ❑ When a number is divided by itself, the answer is 1.

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Strategic Number Counting Instruction

DESCRIPTION: The student is taught explicit number counting strategies for basic addition and subtraction. Those skills are then practiced with a tutor (adapted from Fuchs et al., 2009).

MATERIALS:

- Number-line (attached)
- Number combination (math fact) flash cards for basic addition and subtraction
- *Strategic Number Counting Instruction Score Sheet* (attached)

PREPARATION: The tutor trains the student to use these two counting strategies for addition and subtraction:

ADDITION: The student is given a copy of the appropriate number-line (1-10 or 1-20—see attached). When presented with a two-addend addition problem, the student is taught to start with the larger of the two addends and to 'count up' by the amount of the smaller addend to arrive at the answer to the problem.

SUBTRACTION: The student is given a copy of the appropriate number-line (1-10 or 1-20—see attached).. The student is taught to refer to the first number appearing in the subtraction problem (the minuend) as 'the number you start with' and to refer to the number appearing after the minus (subtrahend) as 'the minus number'. The student is directed to start at the minus number on the number-line and to count up to the starting number while keeping a running tally of numbers counted up on his or her fingers. The final tally of digits separating the minus number and starting number is the answer to the subtraction problem.

INTERVENTION STEPS: For each tutoring session, the tutor follows these steps:

1. *Create Flashcards.* The tutor creates addition and/or subtraction flashcards of problems that the student is to practice. Each flashcard displays the numerals and operation sign that make up the problem but leaves the answer blank.

2. *Review Count-Up Strategies*. At the opening of the session, the tutor asks the student to name the two methods for answering a math fact. The correct student response is 'Know it or count up.' The tutor next has the student describe how to count up an addition problem and how to count up a subtraction problem. Then the tutor gives the student two sample addition problems and two subtraction problems and directs the student to solve each, using the appropriate count-up strategy.

3. *Complete Flashcard Warm-Up*. The tutor reviews addition/subtraction flashcards with the student for three minutes. Before beginning, the tutor reminds the student that, when shown a flashcard, the student should try to 'pull the answer from your head'—but that if the student does not know the answer, he or she should use the appropriate count-up strategy. The tutor then reviews the flashcards with the student. Whenever the student makes an error, the tutor directs the student to use the correct count-up strategy to solve. NOTE: If the student cycles through all cards in the stack before the three-minute period has elapsed, the tutor shuffles the cards and begins again.

   At the end of the three minutes, the tutor counts up the number of cards reviewed and records the number of cards that the student (a) identified from memory, (b) solved using the count-up strategy, and (c) was not able to correctly answer. These totals are recorded on the Strategic *Number Counting Instruction Score Sheet*

4. *Repeat Flashcard Review*. The tutor shuffles the math-fact flashcards, encourages the student to try to beat his or her previous score, and again reviews the flashcards with the student for three minutes. As before, whenever the student makes an error, the tutor directs the student to use the appropriate count-up strategy. Also, if the student completes all cards in the stack with time remaining, the tutor shuffles the stack and continues presenting cards until the time is elapsed.

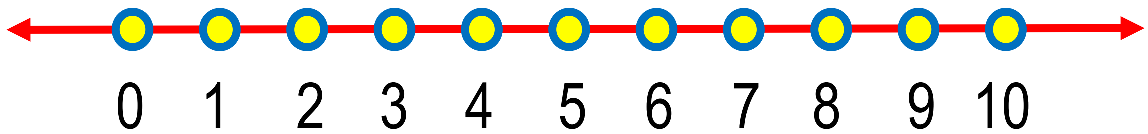
   At the end of the three minutes, the tutor again counts up the number of cards reviewed and records the number of cards that the student (a) identified from memory, (b) solved using the count-up strategy, and (c) was not able to correctly answer. These totals are again recorded on the Strategic *Number Counting Instruction Score Sheet.*

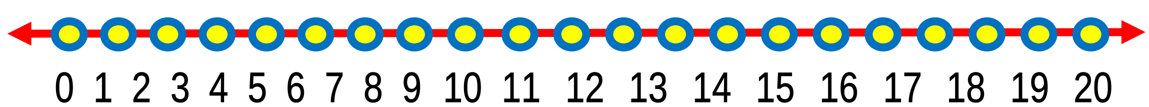
5. *Provide Performance Feedback*. The tutor gives the student feedback about whether (and by how much) the student's performance on the second flashcard trial exceeded the first. The tutor also provides praise if the student beat the previous score or encouragement if the student failed to beat the previous score.

Reference

Fuchs, L. S., Powell, S. R., Seethaler, P. M., Cirino, P. T., Fletcher, J. M., Fuchs, D., & Hamlett, C. L. (2009). The effects of strategic counting instruction, with and without deliberate practice, on number combination skill among students with mathematics difficulties. *Learning and Individual Differences 20*(2), 89-100.

Strategic Number Counting Instruction: Number-Lines





Strategic Number Counting Instruction Score Sheet

Student: ______________________________ Interventionist(s): _________________________________

Directions: During the strategic number counting instruction intervention, use this sheet to tally student responses: Number of Flash-Cards Known From Memory; Number of Flash-Cards Answered Correctly With Count-Up Strategy (with or without assistance); Number of Flash-Cards Unknown or Answered Incorrectly (even with assistance).

Date:________	[Optional] Type/Range of Addition/Subtraction Math-Fact Flash-Cards Reviewed This Session:	
Trial 1: Math Flash-Card Warm-Up: 3 Minutes		
Number of Flash-Cards Known From Memory	Number of Flash-Cards Answered Correctly With Count-Up Strategy	Number of Flash-Cards Unknown or Answered Incorrectly
Trial 2: Math Flash-Card Review: 3 Minutes		
Number of Flash-Cards Known From Memory	Number of Flash-Cards Known From Memory	Number of Flash-Cards Known From Memory

Date:________	[Optional] Type/Range of Addition/Subtraction Math-Fact Flash-Cards Reviewed This Session:	
Trial 1: Math Flash-Card Warm-Up: 3 Minutes		
Number of Flash-Cards Known From Memory	Number of Flash-Cards Answered Correctly With Count-Up Strategy	Number of Flash-Cards Unknown or Answered Incorrectly
Trial 2: Math Flash-Card Review: 3 Minutes		
Number of Flash-Cards Known From Memory	Number of Flash-Cards Known From Memory	Number of Flash-Cards Known From Memory

Student Self-Monitoring of Productivity to Increase Fluency on Math Computation Worksheets

DESCRIPTION: The student monitors and records her or his work production on math computation worksheets on a daily basis—with a goal of improving overall fluency (Maag, Reid, R., & DiGangi, 1993). This intervention can be used with a single student, a small group, or an entire class.

MATERIALS:

- Student self-monitoring audio prompt: Tape / audio file with random tones or dial-style kitchen timer
- Math computation worksheets containing problems targeted for increased fluency
- *Student Speed Check!* recording form (attached)

Preparation: To prepare for the intervention the teacher:

1. *Decides on the Length and Frequency of Each Self-Monitoring Period.* The instructor decides on the length of session and frequency of the student's self-monitoring intervention. NOTE: One good rule of thumb is to set aside at least 10 minutes per day for this or other interventions to promote fluent student retrieval of math facts (Gersten et al., 2009). For example, Mrs. Rilke, a 3rd-grade teacher, decides that her student, Roy, will monitor his productivity on math computation worksheets on a daily basis for 10 minutes per session.

2. *Selects a Math Computation Skill Target*. The instructor chooses one or more problem types that are to appear in intervention worksheets. For example, Mrs. Rilke decides to target two math computation problem-types for Roy: Addition—double-digit plus double-digit with regrouping and Subtraction—double-digit plus double-digit with no regrouping.

3. *Creates Math Computation Worksheets*. When the teacher has chosen the problem types, he or she makes up sufficient equivalent worksheets (with the same number of problems and the same mix of problem-types) to be used across the intervention days. Each worksheet should have enough problems to keep the student busy for the length of time set aside for a self-monitoring intervention session.

   For example, when designing a worksheet, Mrs. Rilke decides to include 15 problems per sheet for her 3rd grade student, to keep Roy busy for the 10 minute daily intervention period. The teacher then goes to the free math worksheet generator at www.interventioncentral.org to create and print off 25 equivalent math worksheets for use across all intervention days (5 days per week for five instructional weeks).

4. *Determines How Many Audio Prompts the Student Will Receive.* This intervention relies on student self-monitoring triggered by audio prompts. Therefore, the teacher must decide on a fixed number of audio prompts the student is to receive per session. NOTE: On the attached *Student Speed Check!* form, space is provided for the student to record productivity for up to five audio prompts per session.

5. *Selects a Method to Generate Random Audio Prompts*. Next, the teacher must decide on how to generate the audio prompts (tones) that drive this intervention. There are two possible choices:

(A) The teacher can develop a tape or audio file that has several random tones spread across the time-span of the intervention session, with the number of tones equaling the fixed number of audio prompts selected for the intervention (see previous step). For example, the instructor may develop a 10-minute tape with five tones randomly sounding at 2 minutes, 3 minutes, 5 minutes, 7 minutes, and 10 minutes.

(B) The instructor may purchase a dial-type kitchen timer. During the intervention period, the instructor turns the dial to a randomly selected number of minutes. When the timer expires and chimes as a student audio prompt, the teacher resets the timer to another random number of minutes and repeats this process until the intervention period is over. Of course, the teacher must ensure that the student receives the same fixed number of audio prompts (e.g., 5) across each intervention session and that all audio prompts are delivered by the conclusion of the timed intervention session. Before each intervention session, the teacher may want to preselect several random time intervals. For example, on a given day, the instructor who wants to include five timer prompts in a 10 minute intervention session may decide to ring the timer at 2 minutes, 3 minutes, 5 minutes, 7 minutes, and 10 minutes. This sequence would then be changed for the next session.

6. *Trains the Student in the Procedures to Self-Monitor Productivity.* The teacher meets with the student to train him or her in the steps of the intervention (described in the next session).

INTERVENTION STEPS: Sessions of the productivity self-monitoring intervention for math computation include these steps:

1. *[Student] Set a Session Computation Goal.* The student looks up the total number of problems completed on his or her most recent timed worksheet and writes that figure into the 'Score to Beat' section of the current day's *Student Speed Check!* form.

2. *[Teacher] Set the Timer or Start the Tape.* The teacher directs the student to begin working on the worksheet and either starts the tape with tones spaced at random intervals or sets a kitchen timer. If using a timer, the teacher randomly sets the timer randomly to a specific number of minutes. When the timer expires and chimes as a student audio prompt, the teacher resets the timer to another random number of minutes and repeats this process until the intervention period is over.

3. *[Student] At Each Tone, Record Problems Completed.* Whenever the student hears an audio prompt or at the conclusion of the timed intervention period, the student pauses to:

    a. circle the problem that he or she is currently working on

    b. count up the number of problems completed since the previous tone (or in the case of the first tone, the number of problems completed since starting the worksheet)

    c. record the number of completed problems next to the appropriate tone interval on the attached *Student Speed Check!* form.

4. [Teacher] *Announce the End of the Intervention Period.* The teacher announces that the intervention period is over and that the student should stop working on the worksheet. NOTE: If a tape or audio file is being used to deliver audio tones, it can contain an announcement stating that the intervention period has ended.

5. *[Student] Tally Day's Performance.* The student adds up the problems completed at the tone-intervals to give a productivity total for the day. The student then compares the current day's figure to that of the previous day to see if he or she was able to beat the previous score. If YES, the student receives praise from the teacher; if NO, the student receives encouragement from the teacher.

References

Maag, J. W., Reid, R., & DiGangi, S. A. (1993). Differential effects of self-monitoring attention, accuracy, and productivity. *Journal of Applied Behavior Analysis, 26*, 329-344.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel,B. (2009). *Assisting students struggling with mathematics: Response to Intervention RtI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sci ences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications/practiceguides/

Student Speed Check!

Student Name: ______________________________ Classroom: ______________

Directions: Use this form to track your speed in completing math worksheets.

Score to Beat: How many problems did I complete at my last session?	_____ Problems
Today's Session:	Date: _________
How many problems did I complete at TONE #1?	_____ Problems
How many more problems did I complete at TONE #2?	_____ Problems
How many more problems did I complete at TONE #3?	_____ Problems
How many more problems did I complete at TONE #4?	_____ Problems
How many more problems did I complete at TONE #5?	_____ Problems
How many more problems did I complete between the final tone and the end of the session?	_____ Problems
TOTAL number of problems completed in this session:	_____ Problems
Did I beat my previous score?	___ Yes ___ No

Score to Beat: How many problems did I complete at my last session?	_____ Problems
Today's Session:	Date: _________
How many problems did I complete at TONE #1?	_____ Problems
How many more problems did I complete at TONE #2?	_____ Problems
How many more problems did I complete at TONE #3?	_____ Problems
How many more problems did I complete at TONE #4?	_____ Problems
How many more problems did I complete at TONE #5?	_____ Problems
How many more problems did I complete between the final tone and the end of the session?	_____ Problems
TOTAL number of problems completed in this session:	_____ Problems
Did I beat my previous score?	___ Yes ___ No

Increase Student Math Success with Customized Math Self-Correction Checklists

DESCRIPTION: The teacher analyzes a particular student's pattern of errors commonly made when solving a math algorithm (on either computation or word problems) and develops a brief error self-correction checklist unique to that student. The student then uses this checklist to self-monitor—and when necessary correct—his or her performance on math worksheets before turning them in.

MATERIALS:

- Customized student math error self-correction checklist (described below)
- Worksheets or assignments containing math problems matched to the error self-correction checklist

INTERVENTION STEPS: The intervention with customized math error self-correction checklists includes these steps (adapted from Dunlap & Dunlap, 1989; Uberti et al., 2004):

1. *Develop the Checklist*. The teacher draws on multiple sources of data available in the classroom to create a list of errors that the student commonly makes on a specific type of math computation or word problem. Good sources of information for analyzing a student's unique pattern of math-related errors include review of completed worksheets and other work products, interviewing the student, asking the student to solve a math problem using a 'think aloud' approach to walk through the steps of an algorithm, and observing the student completing math problems in a cooperative learning activity with other children.

   Based on this error analysis, the teacher creates a short (4-to-5 item) student self-correction checklist that includes the most common errors made by that student. Items on the checklist are written in the first person and when possible are stated as 'replacement' or goal behaviors. This checklist might include steps in an algorithm that challenge the student (e.g., "I underlined all numbers at the top of the subtraction problem that were smaller than their matching numbers at the bottom of the problem") as well as goals tied to any other errors that impede math performance (e.g., "I wrote all numbers carefully so that I could read them easily and not mistake them for other numbers").

   NOTE: To reduce copying costs, the teacher can laminate the self-correction checklist and provide the student with an erasable marker to allow for multiple re-use of the form.

2. *Introduce the Checklist.* The teacher shows the student the self-correction checklist customized for that student. The teacher states that the student is to use the checklist to check his or her work before turning it in so that the student can identify and correct the most common errors.

3. *Prompt the Student to Use the Checklist to Evaluate Each Problem.* The student is directed to briefly review all items on the checklist before starting any worksheet or assignment containing the math problems that it targets.

   When working on the math worksheet or assignment, the student uses the checklist after *every* problem to check his or her work—marking each checklist item with a plus sign ('+') if correctly followed or a minus sign ('-') if not correctly followed. If any checklist item receives a minus rating, the student is directed to leave the original solution to the problem untouched, to solve the problem again, and again to use the checklist to check the work. Upon finishing the assignment, the student turns it in, along with the completed self-correction checklists.

4. *Provide Performance Feedback, Praise, and Encouragement*. Soon after the student submits any math worksheets associated with the intervention, the teacher should provide him or her with timely feedback about errors, praise for correct responses, and encouragement to continue to apply best effort.

5. *[OPTIONAL] Provide Reinforcement for Checklist Use*. If the student appears to need additional incentives to increase motivation for the intervention, the teacher can assign the student points for intervention compliance: (1) the student earns one point on any assignment for each correct answer, and (2) the student earns an additional point for each problem on which the student committed none of the errors listed on the self-correction checklist. The student is allowed to collect points and to redeem them for privileges or other rewards in a manner to be determined by the teacher.

6. *Fade the Intervention*. The error self-correction checklist can be discontinued when the student is found reliably to perform on the targeted math skill(s) at a level that the teacher defines as successful (e.g., 90 percent success or greater).

Reference

Dunlap, L. K., & Dunlap, G. (1989). A self-monitoring package for teaching subtraction with regrouping to students with learning disabilities. *Journal of Applied Behavior Analysis, 229*, 309-314.

Uberti, H. Z., Mastropieri, M. A., & Scruggs, T. E. (2004). Check it off: Individualizing a math algorithm for students with disabilities via self-monitoring checklists. *Intervention in School and Clinic, 39*(5), 269-275.

SAMPLE: Math Self-Correction Checklist

Student Name: ______________________________ Date: ______________________________

Rater: Student ______________________________ Classroom: ______________________________

Directions: To the Student: BEFORE YOU START: Look at each of these goals for careful math work before beginning your assignment. AFTER EACH PROBLEM: Stop and rate YES or NO whether you performed each goal correctly.

	Problem#1	Problem#2	Problem#3	Problem#4	Problem#5
I underlined all numbers at the top of the subtraction problem that were smaller than their matching numbers at the bottom of the problem. Did the student succeed in this behavior goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N
I wrote all numbers carefully so that I could read them easily and not mistake them for other numbers. Did the student succeed in this behavior goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N
I lined up all numbers in the right place-value columns. Did the student succeed in this behavior goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N
I rechecked all of my answers. Did the student succeed in this behavior goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N

Math Self-Correction Checklist

Student Name: ______________________________ Date: ______________________________

Rater: Student ______________________________ Classroom: ______________________________

Directions: To the Student: BEFORE YOU START: Look at each of these goals for careful math work before beginning your assignment. AFTER EACH PROBLEM: Stop and rate YES or NO whether you performed each goal correctly.

	Problem#1	Problem#2	Problem#3	Problem#4	Problem#5
______________________________ Did the student succeed in this math goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N
______________________________ Did the student succeed in this math goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N
______________________________ Did the student succeed in this math goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N
______________________________ Did the student succeed in this math goal? ❑ YES ❑ NO	__Y __N	__Y __N	__Y __N	__Y __N	__Y __N

Math Review: Promote Mastery of Math Facts Through Incremental Rehearsal

Incremental rehearsal builds student fluency in basic math facts ('arithmetic combinations') by pairing unknown computation items with a steadily increasing collection of known items. This intervention makes use of repeated, or massed, practice to promote fluency and guarantees that the student will experience a high rate of success..

Materials

- Index cards and pen

Steps to Implementing This Intervention

In preparation for this intervention:

1. The tutor first writes down on an index card in ink each math fact that a student is expected to master-but without the answer. NOTE: Educators can use the A-Plus Math Flashcard Creator, an on-line application, to make and print flashcards in addition, subtraction, multiplication, and division. The web address for the flashcard creator is: *http://www.aplusmath.com/Flashcards/Flashcard_Creator.html*

2. The tutor reviews the collection of math-fact cards with the student. Any of the math facts that the student can orally answer correctly within two seconds are considered to be known problems and are separated into one pile. Math facts that the student cannot yet answer correctly within two seconds are considered 'unknown' and collected in a second pile -- the 'unknown facts' deck.

3. The tutor next randomly selects 9 cards from the pile of known math facts and sets this subset of cards aside as the 'known facts' deck. The rest of the pile of cards containing known math facts is put away ('discard deck'), not to be used further in this intervention.

During the intervention:

The tutor follows an incremental-rehearsal sequence each day when working with the student:

1. First, the tutor takes a single card from the 'unknown facts' deck. The tutor reads the math fact on the card aloud, provides the answer, and prompts the student to read off and answer the same unknown problem.

2. Next the tutor takes one math fact from the 'known facts' deck and pairs it with the unknown problem. When shown the two problems in sequence, the student is asked during the presentation of each math fact to read off the problem and answer it. The student is judged to be successful on a problem if he or she orally provides the correct answer to that problem within 2 seconds. If the student commits an error on any card or hesitates for longer than two seconds, the tutor reads the math fact on the card aloud, gives the answer, then prompts the

student to read off the same unknown problem and provide the answer. This review sequence continues until the student answers all cards within two seconds without errors.

3. The tutor then repeats the sequence--taking yet another problem from the 'known facts' deck to add to the expanding collection of math facts being reviewed ('review deck'). Each time, the tutor prompts the student to read off and answer the whole series of math facts in the review deck, beginning with the unknown fact and then moving through the growing series of known facts that follow it.

4. When the review deck has expanded to include one 'unknown' math fact followed by nine 'known' math facts (a ratio of 90 percent 'known' material to 10 percent 'unknown' material), the last 'known' math fact that was added to the student's review deck is discarded (put away with the 'discard deck'). The previously 'unknown' math fact that the student has just successfully practiced in multiple trials is now treated as a 'known' math fact and is included as the first item in the nine-card 'known facts' deck for future drills.

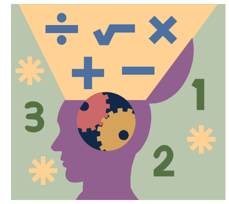
5. The student is then presented with a new math fact to answer, taken from the 'unknown facts' deck. With each new 'unknown' math fact, the review sequence is again repeated as described above until the 'unknown' math fact is grouped incrementally with nine math facts from the 'known facts' deck—and on and on.

Daily review sessions are discontinued either when time runs out or when the student answers an 'unknown' math fact incorrectly three times.

Reference

Burns, M. K. (2005). Using incremental rehearsal to increase fluency of single-digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children, 28*, 237-249.

Math Computation: Increase Accuracy By Intermixing Easy and Challenging Problems



Teachers can improve accuracy and positively influence the attitude of students when completing math-fact worksheets by intermixing 'easy' problems among the 'challenging' problems. Research shows that students are more motivated to complete computation worksheets when they contain some very easy problems interspersed among the more challenging items.

Materials

- Math computation worksheets & answer keys with a mixture of difficult and easy problems

Steps to Implementing This Intervention

1. The teacher first identifies one or more 'challenging' problem-types that are matched to the student's current math-computation abilities (e.g., multiplying a 2-digit number by a 2-digit number with regrouping).

2. The teacher next identifies an 'easy' problem-type that the students can complete very quickly (e.g., adding or subtracting two 1-digit numbers).

3. The teacher then creates a a series of student math computation worksheets with 'easy' computation problems interspersed at a fixed rate among the 'challenging' problems. (NOTE: Instructions are included below for creating interspersal worksheets using a free online application from *www.interventioncentral.org*.)

    - If the student is expected to complete the worksheet independently as seat work or homework, 'challenging' and 'easy' problems should be interspersed at a 1:1 ratio (that is, every 'challenging' problem in the worksheet is followed by an 'easy' problem).

    - If the student is to have the problems read aloud and then asked to solve the problems mentally and write down only the answer, the items should appear on the worksheet at a ratio of 3:1 (that is, every third 'challenging' problem is followed by an 'easy' one).

Directions for On-Line Creation of Worksheets With a Mix of Easy and Challenging Computation Problems ('Interspersal Worksheets')

By following the directions below, teachers can use a free on-line Math Worksheet Generator to create computation worksheets with easy problems interspersed among more challenging ones:

- The teacher goes to the following URL for the Math Worksheet Generator: http://www.interventioncentral.org/htmdocs/tools/mathprobe/allmult.php

- Displayed on that Math Worksheet Generator web page is a series of math computation goals for addition, subtraction, multiplication, and division. Teachers can select up to five different problem types to appear on a student worksheet. Each problem type is selected by clicking on the checkbox next to it.

- It is simple to create a worksheet with a 1:1 ratio of challenging and easy problems (that is, with an easy problem following every challenging problem). First, the teacher clicks the checkbox next to an 'easy' problem type that the student can compute very quickly (e.g., adding or subtracting two 1-digit numbers). Next the teacher selects a 'challenging' problem type that is instructionally appropriate for the student (e.g., multiplying a 2-digit number by a 2-digit number with regrouping). Then the teacher clicks the 'Multiple Skill Computation Probe' button. The computer program will then automatically create a student computation worksheet and teacher answer key with alternating easy and challenging problems.

- It is also no problem to create a worksheet with a higher (e.g., 2:1, 3:1, or 4:1) ratio of challenging problems to easy problems. The teacher first clicks the checkbox next to an 'easy' problem type that the student can compute very quickly (e.g., adding or subtracting two 1-digit numbers). The teacher then selects up to four different challenging problem types that are instructionally appropriate to the student. Depending on the number of challenging problem-types selected, when the teacher clicks the 'Multiple Skill Computation Probe' button, the computer program will create a student computation worksheet and teacher answer key that contain 2 (or 3 or 4) challenging problems for every easy problem.

Because the computer program generates new worksheets each time it is used, the teacher can enter the desired settings and –in one sitting-- create and print off enough worksheets and answer keys to support a six- or eight-week intervention.

Reference

Hawkins, J., Skinner, C. H., & Oliver, R. (2005). The effects of task demands and additive interspersal ratios on fifth-grade students' mathematics accuracy. *School Psychology Review, 34,* 543-555.

Peer Tutoring in Math Computation with Constant Time Delay

DESCRIPTION: This intervention employs students as reciprocal peer tutors to target acquisition of basic math facts (math computation) using constant time delay (Menesses & Gresham, 2009; Telecsan, Slaton, & Stevens, 1999). Each tutoring 'session' is brief and includes its own progress-monitoring component--making this a convenient and time-efficient math intervention for busy classrooms.

MATERIALS:

Student Packet: A work folder is created for each tutor pair. The folder contains:

- ❑ 10 math fact cards with equations written on the front and correct answer appearing on the back. NOTE: The set of cards is replenished and updated regularly as tutoring pairs master their math facts.
- ❑ Progress-monitoring form for each student.
- ❑ Pencils.

PREPARATION: To prepare for the tutoring program, the teacher selects students to participate and trains them to serve as tutors.

Select Student Participants. Students being considered for the reciprocal peer tutor program should at minimum meet these criteria (Telecsan, Slaton, & Stevens, 1999, Menesses & Gresham, 2009):

- ❑ Is able and willing to follow directions;
- ❑ Shows generally appropriate classroom behavior;
- ❑ Can attend to a lesson or learning activity for at least 20 minutes.
- ❑ Is able to name all numbers from 0 to 18 (if tutoring in addition or subtraction math facts) and name all numbers from 0 to 81 (if tutoring in multiplication or division math facts).
- ❑ Can correctly read aloud a sampling of 10 math-facts (equation plus answer) that will be used in the tutoring sessions. (NOTE: The student does not need to have memorized or otherwise mastered these math facts to participate—just be able to read them aloud from cards without errors).
- ❑ [To document a deficit in math computation] When given a two-minute math computation probe to complete independently, computes fewer than 20 correct digits (Grades 1-3) or fewer than 40 correct digits (Grades 4 and up) (Deno & Mirkin, 1977).

NOTE: Teachers may want to use the attached *Reciprocal Peer Tutoring in Math Computation: Teacher Nomination Form* to compile a list of students who would be suitable for the tutoring program.

Train the Student Tutors. Student tutors are trained through explicit instruction (Menesses & Gresham, 2009) with the teacher clearly explaining the tutoring steps, demonstrating them, and then having the students practice the steps with performance feedback and encouragement from the teacher. The teacher also explains, demonstrates, and observes students practice the progress-monitoring component of the program. (NOTE: Teachers can find a handy listing of all the tutoring steps in which students are to be trained on the attached form *Peer Tutoring in Math*

Computation with Constant Time Delay: Integrity Checklist. This checklist can also be used to evaluate the performance of students to determine their mastery of the tutoring steps during practice sessions with the teacher.)

When students have completed their training, the teacher has each student role-play the tutor with the teacher assuming the role of tutee. The tutor-in-training works through the 3-minute tutoring segment and completes the follow-up progress-monitoring activity. The teacher then provides performance feedback. The student is considered to be ready to tutor when he or she successfully implements all steps of the intervention (100% accuracy) on three successive training trials (Menesses & Gresham, 2009).

INTERVENTION STEPS: Students participating in the tutoring program meet in a setting in which their tutoring activities will not distract other students. The setting is supervised by an adult who monitors the students and times the tutoring activities. These are the steps of the tutoring intervention:

1. Complete the Tutoring Activity. In each tutoring pair, one of the students assumes the role of tutor. The supervising adult starts the timer and says 'Begin'; after 3 minutes, the adult stops the timer and says 'Stop'.

   While the timer is running, the tutor follows this sequence:

   a. *Presents Cards*. The tutor presents each card to the tutee for 3 seconds.
   b. *Provides Tutor Feedback*. [When the tutee responds correctly] The tutor acknowledges the correct answer and presents the next card.

      [When the tutee does not respond within 3 seconds or responds incorrectly] The tutor states the correct answer and has the tutee repeat the correct answer. The tutor then presents the next card.
   c. *Provides Praise*. The tutor praises the tutee immediately following correct answers.
   d. *Shuffles Cards*. When the tutor and tutee have reviewed all of the math-fact carts, the tutor shuffles them before again presenting cards.
   e. *Continues to the Timer*. The tutor continues to presents math-fact cards for tutee response until the timer rings.
2. Assess the Progress of the Tutee. The tutor concludes each 3-minute tutoring session by assessing the number of math facts mastered by the tutee. The tutor follows this sequence:

   a. *Presents Cards.* The tutor presents each card to the tutee for 3 seconds.
   b. *Remains Silent*. The tutor does not provide performance feedback or praise to the tutee, or otherwise talk during the assessment phase.
   c. *Sorts Cards*. Based on the tutee's responses, the tutor sorts the math-fact cards into 'correct' and 'incorrect' piles.

d. *Counts Cards and Records Totals*. The tutor counts the number of cards in the 'correct' and 'incorrect' piles and records the totals on the tutee's progress-monitoring chart.

3. Switch Roles. After the tutor has completed the 3-minute tutoring activity and assessed the tutee's progress on math facts, the two students reverse roles. The new tutor then implements steps 2 and 3 described above with the new tutee.

4. Conduct Tutoring Integrity Checks and Monitor Student Performance. As the student pairs complete the tutoring activities, the supervising adult monitors the integrity with which the intervention is carried out. At the conclusion of the tutoring session, the adult gives feedback to the student pairs, praising successful implementation and providing corrective feedback to students as needed. NOTE: Teachers can use the attached form *Peer Tutoring in Math Computation with Constant Time Delay: Integrity Checklist* to conduct integrity checks of the intervention and student progress-monitoring components of the math peer tutoring.

   The adult supervisor also monitors student progress. After each student pair has completed one tutoring cycle and assessed and recorded their progress, the supervisor reviews the score sheets. If a student has successfully answered all 10 math fact cards three times in succession, the supervisor provides that student's tutor with a new set of math flashcards.

References

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Reciprocal Peer Tutoring in Math Computation: Teacher Nomination Form

Teacher: ____________________________ Classroom: ________________ Date: __________________

Directions: Select students in your class that you believe would benefit from participation in a peer tutoring program to boost math computation skills. Write the names of your student nominees in the space provided below. Remember, students who are considered for the peer tutoring program should—*at minimum*—meet these criteria:

- Show generally appropriate classroom behaviors and follow directions.
- Can pay attention to a lesson or learning activity for at least 20 minutes.
- Are able to wait appropriately to hear the correct answer from the tutor if the student does not know the answer.
- When given a two-minute math computation probe to complete independently, computes fewer than 20 correct digits (Grades 1-3) or fewer than 40 correct digits (Grades 4 and up) (Deno & Mirkin, 1977).
- Can name all numbers from 0 to 18 (if tutoring in addition or subtraction math facts) and name all numbers from 0 to 81 (if tutoring in multiplication or division math facts).
- Can correctly read aloud a sampling of 10 math-facts (equation plus answer) that will be used in the tutoring sessions. (NOTE: The student does not need to have memorized or otherwise mastered these math facts to participate—just be able to read them aloud from cards without errors).

Number	Student Name	NOTES
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		

Peer Tutoring in Math Computation with Constant Time Delay: Integrity Checklist			
Tutoring Session: Intervention Phase			
Directions: Observe the tutor and tutee for a full intervention session. Use this checklist to record whether each of the key steps of the intervention were correctly followed.			
Correctly Carried Out?	Step	Tutor Action	NOTES
___ Y ___ N	1.	**Promptly Initiates Session.** At the start of the timer, the tutor immediately presents the first math-fact card.	
___ Y ___ N	2.	**Presents Cards.** The tutor presents each card to the tutee for 3 seconds.	
___ Y ___ N	3.	**Provides Tutor Feedback.** [When the tutee responds correctly] The tutor acknowledges the correct answer and presents the next card. [When the tutee does not respond within 3 seconds or responds incorrectly] The tutor states the correct answer and has the tutee repeat the correct answer. The tutor then presents the next card.	
___ Y ___ N	4.	**Provides Praise.** The tutor praises the tutee immediately following correct answers.	
___ Y ___ N	5.	**Shuffles Cards.** When the tutor and tutee have reviewed all of the math-fact carts, the tutor shuffles them before again presenting cards.	
___ Y ___ N	6.	**Continues to the Timer.** The tutor continues to presents math-fact cards for tutee response until the timer rings.	

Tutoring Session: Assessment Phase			
Directions: Observe the tutor and tutee during the progress-monitoring phase of the session. Use this checklist to record whether each of the key steps of the assessment were correctly followed.			
Correctly Carried Out?	Step Tutor	Action	NOTES
___ Y ___ N	1.	**Presents Cards.** The tutor presents each card to the tutee for 3 seconds.	
___ Y ___ N	2.	**Remains Silent.** The tutor does not provide performance feedback or praise to the tutee, or otherwise talk during the assessment phase.	
___ Y ___ N	3.	**Sorts Cards.** The tutor sorts cards into 'correct' and 'incorrect' piles based on the tutee's responses.	
___ Y ___ N	4.	**Counts Cards and Records Totals.** The tutor counts the number of cards in the 'correct' and 'incorrect' piles and records the totals on the tutee's progress-monitoring chart.	

Math Tutoring: Score Sheet

Tutor 'Coach': ________________________ Tutee 'Player': ________________________

Directions to the Tutor: Write down the number of math-fact cards that your partner answered *correctly* and the number answered *incorrectly*.

Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:
Date:	Cards Correct:	Cards Incorrect:

Applied Math Problems: Using Question-Answer Relationships (QARs) to Interpret Math Graphics

Students must be able to correctly interpret math graphics in order to correctly answer many applied math problems. Struggling learners in math often misread or misinterpret math graphics. For example, students may:

- overlook important details of the math graphic.
- treat irrelevant data on the math graphic as 'relevant'.
- fail to pay close attention to the question before turning to the math graphic to find the answer
- not engage their prior knowledge both to extend the information on the math graphic and to act as a possible 'reality check' on the data that it presents.
- expect the answer to be displayed in plain sight on the math graphic, when in fact the graphic may require that readers first to interpret the data, then to plug the data into an equation to solve the problem.

Teachers need an instructional strategy to encourage students to be more savvy interpreters of graphics in applied math problems. One idea is to have them apply a reading comprehension strategy, Question-Answer Relationships (QARs) as a tool for analyzing math graphics. The four QAR question types (Raphael, 1982, 1986) are as follows:

- RIGHT THERE questions are fact-based and can be found in a single sentence, often accompanied by 'clue' words that also appear in the question.
- THINK AND SEARCH questions can be answered by information in the text--but require the scanning of text and the making of connections between disparate pieces of factual information found in different sections of the reading.
- AUTHOR AND YOU questions require that students take information or opinions that appear in the text and combine them with the reader's own experiences or opinions to formulate an answer.
- ON MY OWN questions are based on the students' own experiences and do not require knowledge of the text to answer.

Steps to Implementing This Intervention

Teachers use a 4-step instructional sequence to teach students to use Question-Answer Relationships (QARs) to better interpret math graphics:

1. Step 1: Distinguishing Among Different Kinds of Graphics

   Students are first taught to differentiate between five common types of math graphics: table (grid with information contained in cells), chart (boxes with possible connecting lines or arrows), picture (figure with labels), line graph, bar graph.

   Students note significant differences between the various types of graphics, while the teacher records those observations on a wall chart. Next students are shown examples of graphics and directed to identify the general graphic type (table, chart, picture, line graph, bar graph) that each sample represents.

As homework, students are assigned to go on a ‘graphics hunt’, locating graphics in magazines and newspapers, labeling them, and bringing them to class to review.

2. Interpreting Information in Graphics

Over several instructional sessions, students learn to interpret information contained in various types of math graphics. For these activities, students are paired off, with stronger students matched with less strong ones.

The teacher sets aside a separate session to introduce each of the graphics categories. The presentation sequence is ordered so that students begin with examples of the most concrete graphics and move toward the more abstract. The graphics sequence in order of increasing difficulty is: Pictures > tables > bar graphs > charts > line graphs.

At each session, student pairs examine examples of graphics from the category being explored that day and discuss questions such as: “What information does this graphic present? What are strengths of this type of graphic for presenting data? What are possible weaknesses?” Student pairs record their findings and share them with the large group at the end of the session.

3. Linking the Use of Question-Answer Relations (QARs) to Graphics

In advance of this lesson, the teacher prepares a series of data questions and correct answers. Each question and answer is paired with a math graphic that contains information essential for finding the answer.

At the start of the lesson, students are each given a set of 4 index cards with titles and descriptions of each of the 4 QAR questions: RIGHT THERE, THINK AND SEARCH, AUTHOR AND YOU, ON MY OWN. (TMESAVING TIP: Students can create their own copies of these QAR review cards as an in-class activity.)

Working first in small groups and then individually, students read each teacher-prepared question, study the matching graphic, and ‘verify’ the provided answer as correct. They then identify the type of question being posed in that applied problem, using their QAR index cards as a reference.

4. Using Question-Answer Relationships (QARs) Independently to Interpret Math Graphics

Students are now ready to use the QAR strategy independently to interpret graphics. They are given a laminated card as a reference with 6 steps to follow whenever they attempt to solve an applied problem that includes a math graphic:

- ✓ Read the question,
- ✓ Review the graphic,
- ✓ Reread the question,
- ✓ Choose a Question-Answer Relationship that matches the question in the applied problem
- ✓ Answer the question, and
- ✓ Locate the answer derived from the graphic in the answer choices offered.

Students are strongly encouraged NOT to read the answer choices offered on a multiple-choice item until they have first derived their own answer—to prevent those choices from short-circuiting their inquiry.

References

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Math Computation: Increase Accuracy and Productivity Rates Via Self-Monitoring and Performance Feedback

Students can improve both their accuracy and fluency on math computation worksheets by independently self-monitoring their computation speed, charting their daily progress, and earning rewards for improved performance.

Materials

- Collection of student math computation worksheets & matching answer keys (NOTE: Educators can use a free online application to create math computation worksheets and answer keys at http://www.interventioncentral.org/htmdocs/tools/mathprobe/addsing.php)

- Student self-monitoring chart

Steps to Implementing This Intervention

In preparation for this intervention:

- the teacher selects one or more computation problem types that the student needs to practice. Using that set of problem types as a guide, the teacher creates a number of standardized worksheets with similar items to be used across multiple instructional days. (A Math Worksheet Generator that will create these worksheets automatically can be accessed at http://www.interventioncentral.org).

- the teacher prepares a progress-monitoring chart. The vertical axis of the chart extends from 0 to 100 and is labeled ‘Correct Digits’ The horizontal axis of the chart is labeled ‘Date’.

- the teacher creates a menu of rewards that the student can choose from on a given day if the student was able to exceed his or her previously posted computation fluency score.

At the start of the intervention, the teacher meets with the student. The teacher shows the student a sample math computation worksheet and answer key. The teacher tells the student that the student will have the opportunity to complete similar math worksheets as time drills and chart the results. The student is told that he or she will win a reward on any day when the student’s number of correctly computed digits on the worksheet exceeds that of the previous day.

During each day of the intervention:

1. The student is given one of the math computation worksheets previously created by the teacher, along with an answer key. The student first consults his or her progress-monitoring chart and notes the most recent charted computation fluency score previously posted. The student is encouraged to try to exceed that score.

2. When the intervention session starts, the student is given a pre-selected amount of time (e.g., 5 minutes) to complete as many problems on the computation worksheet as possible. The student sets a timer for the allocated time and works on the computation sheet until the timer rings.

3. The student then uses the answer key to check his or her work, giving credit for each correct digit in an answer. (A 'correct digits' is defined as a digit of the correct value that appears in the correct place-value location in an answer. In this scoring method, students can get partial credit even if some of the digits in an answer are correct and some are incorrect.).

4. The student plots his or her computational fluency score on the progress-monitoring chart and writes the current date at the bottom of the chart below the plotted data point. The student is allowed to select a choice from the reward menu if he or she exceeds his or her most recent, previously posted fluency score.

References

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Shimabukuro, S. M., Prater, M. A., Jenkins, A., & Edelen-Smith, P. (1999). The effects of self-monitoring of academic performance on students with learning disabilities and ADD/ADHD. *Education and Treatment of Children, 22*, 397-414.

Combining Cognitive & Metacognitive Strategies to Assist Students With Mathematical Problem Solving

Solving an advanced math problem independently requires the coordination of a number of complex skills. The student must have the capacity to reliably implement the specific steps of a particular problem-solving process, or cognitive strategy. At least as important, though, is that the student must also possess the necessary metacognitive skills to analyze the problem, select an appropriate strategy to solve that problem from an array of possible alternatives, and monitor the problem-solving process to ensure that it is carried out correctly.

The following strategies combine both cognitive and metacognitive elements (Montague, 1992; Montague & Dietz, 2009). First, the student is taught a 7-step process for attacking a math word problem (cognitive strategy). Second, the instructor trains the student to use a three-part self-coaching routine for each of the seven problem-solving steps (metacognitive strategy).

In the cognitive part of this multi-strategy intervention, the student learns an explicit series of steps to analyze and solve a math problem. Those steps include:

1. Reading the problem. The student reads the problem carefully, noting and attempting to clear up any areas of uncertainly or confusion (e.g., unknown vocabulary terms).
2. Paraphrasing the problem. The student restates the problem in his or her own words.
3. 'Drawing' the problem. The student creates a drawing of the problem, creating a visual representation of the word problem.
4. Creating a plan to solve the problem. The student decides on the best way to solve the problem and develops a plan to do so.
5. Predicting/Estimating the answer. The student estimates or predicts what the answer to the problem will be. The student may compute a quick approximation of the answer, using rounding or other shortcuts.
6. Computing the answer. The student follows the plan developed earlier to compute the answer to the problem.
7. Checking the answer. The student methodically checks the calculations for each step of the problem. The student also compares the actual answer to the estimated answer calculated in a previous step to ensure that there is general agreement between the two values.

The metacognitive component of the intervention is a three-part routine that follows a sequence of 'Say', 'Ask, 'Check'. For each of the 7 problem-solving steps reviewed above:

- The student first self-instructs by stating, or 'saying', the purpose of the step ('Say').
- The student next self-questions by 'asking' what he or she intends to do to complete the step ('Ask').
- The student concludes the step by self-monitoring, or 'checking', the successful completion of the step ('Check').

While the Say-Ask-Check sequence is repeated across all 7 problem-solving steps, the actual content of the student self-coaching comments changes across the steps.

Table 1 shows how each of the steps in the word problem cognitive strategy is matched to the three-part Say-Ask-Check sequence:

Table 1: 'Say-Ask-Check' Metacognitive Prompts Tied to a Word-Problem Cognitive Strategy (Montague, 1992)		
Cognitive Strategy Step	Metacognitive 'Say-Ask-Check' Prompt Targets	Sample Metacognitive 'Say-Ask-Check' Prompts
1. Read the problem.	'Say' (Self-Instruction) Target: *The student reads and studies the problem carefully before proceeding.* *'Ask' (Self-Question) Target: Does the student fully understand the problem?* *'Check' (Self-Monitor) Target: Proceed only if the problem is understood.*	Say: "I will read the problem. I will reread the problem if I don't understand it." Ask: "Now that I have read the problem, do I fully understand it?" Check: "I understand the problem and will move forward."
2. Paraphrase the problem.	'Say' (Self-Instruction) Target: *The student restates the problem in order to demonstrate understanding.* *'Ask' (Self-Question) Target: Is the student able to paraphrase the problem?* *'Check' (Self-Monitor) Target: Ensure that any highlighted key words are relevant to the question.*	Say: "I will highlight key words and phrases that relate to the problem question." "I will restate the problem in my own words." Ask: "Did I highlight the most important words or phrases in the problem?" Check: "I found the key words or phrases that will help to solve the problem."
3. 'Draw' the problem.	'Say' (Self-Instruction) Target: *The student creates a drawing of the problem to consolidate understanding.* *'Ask' (Self-Question) Target: Is there a match between the drawing and the problem?* *'Check' (Self-Monitor) Target: The drawing includes in visual form the key elements of the math problem.*	Say: "I will draw a diagram of the problem." Ask: "Does my drawing represent the problem?" Check: "The drawing contains the essential parts of the problem."
4. Create a plan to solve the problem.	'Say' (Self-Instruction) Target: *The student generates a plan to solve the problem.* *'Ask' (Self-Question) Target: What plan will help the student to solve this problem?* *'Check' (Self-Monitor) Target: The plan is appropriate to solve the problem.*	Say: "I will make a plan to solve the problem." Ask: "What is the first step of this plan? What is the next step of the plan?" Check: "My plan has the right steps to solve the problem."
5. Predict/estimate the	'Say' (Self-Instruction) Target: *The student uses estimation or other strategies to predict or*	Say: "I will estimate what the answer will be."

Answer.	*estimate the answer.* ***'Ask' (Self-Question) Target:*** *What estimating technique will the student use to predict the answer?* ***'Check' (Self-Monitor) Target:*** *The predicted/estimated answer used all of the essential problem information.*	Ask: "What numbers in the problem should be used in my estimation?" Check: "I did not skip any important information in my estimation."
6. Compute the answer.	'Say' (Self-Instruction) Target: *The student follows the plan to compute the solution to the problem.* ***'Ask' (Self-Question) Target:*** *Does the answer agree with the estimate?* ***'Check' (Self-Monitor) Target:*** *The steps in the plan were followed and the operations completed in the correct order.*	Say: "I will compute the answer to the problem." Ask: "Does my answer sound right?" "Is my answer close to my estimate?" Check: "I carried out all of the operations in the correct order to solve this problem."
7. Check the answer.	'Say' (Self-Instruction) Target: *The student reviews the computation steps to verify the answer.* ***'Ask' (Self-Question) Target:*** *Did the student check all the steps in solving the problem and are all computations correct?* ***'Check' (Self-Monitor) Target:*** *The problem solution appears to have been done correctly.*	Say: "I will check the steps of my answer." Ask: "Did I go through each step in my answer and check my work?" Check: ""

Students will benefit from close teacher support when learning to combine the 7-step cognitive strategy to attack math word problems with the iterative 3-step metacognitive Say-Ask-Check sequence. Teachers can increase the likelihood that the student will successfully acquire these skills by using research-supported instructional practices (Burns, VanDerHeyden, & Boice, 2008), including:

- Verifying that the student has the necessary foundation skills to solve math word problems
- Using explicit instruction techniques to teach the cognitive and metacognitive strategies
- Ensuring that all instructional tasks allow the student to experience an adequate rate of success
- Providing regular opportunities for the student to be engaged in active accurate academic responding
- Offering frequent performance feedback to motivate the student and shape his or her learning.

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RTI-Ready Methods to Monitor Student Academics
Math: Early Math Fluency

☐ Quantity Discrimination Fluency	⌛: 1 minute	Administration: 1:1
Description: The student is given a sheet with number pairs. For each number pair, the student must name the larger of the two numbers.		
Where to get materials: • AimsWeb http://www.aimsweb.com/ • Intervention Central http://www.interventioncentral.org (Numberfly Early Math Fluency Probe Creator)		

☐ Missing Number Fluency	⌛: 1 minute	Administration: 1:1
Description: The student is given a sheet containing numerous sets of 3 or 4 sequential numbers. For each number series, one of the numbers is missing. The student must name the missing number.		
Where to get materials: • AimsWeb http://www.aimsweb.com/ • Intervention Central http://www.interventioncentral.org (Numberfly Early Math Fluency Probe Creator)		

☐ Number Identification Fluency	⌛: 1 minute	Administration: 1:1
Description: The student is given a sheet with numbers in random order. The student gives the name of each number.		
Where to get materials: • AimsWeb http://www.aimsweb.com/ • Intervention Central http://www.interventioncentral.org (Numberfly Early Math Fluency Probe Creator)		

☐ Oral Counting Fluency	⌛: 1 minute	Administration: 1:1
Description: The student counts aloud as many words in sequence as possible, starting from zero or one.		
Where to get materials: • The student does not require materials for this assessment. The examiner can make a sheet with numbers listed sequentially from 0-100 to record those numbers that the student can recite in sequence.		

Math: Computation

☐ Math Computation Fluency ⌛: 2 minutes Administration: Group
Description: The student is given a worksheet with single-skill or mixed-skill math computation problems. The student works independently to complete as many problems as possible. The student receives credit for each correct digit appearing in his or her answer.
Where to get materials: • AimsWeb http://www.aimsweb.com/ • Intervention Central http://www.interventioncentral.org (Math Worksheet Generator) • SuperKids http://www.superkids.com/aweb/tools/math/ (This website allows you to create math computation worksheets for more advanced areas such as fractions, percentages, decimals, and more)

Math: Applied Problems

☐ Math Concepts & Applications ⌛: 6-8 minutes Administration: Group
Description: Students are given assessment booklets with a mix of applied problem types appropriate to that grade level. (Assessments are available for grades 2-6). A mix of applied problems is included in each assessment, sampling the typical math curriculum for the student's grade (e.g., money skills, time-telling, etc.)
Where to get materials: • MBSP: Monitoring Basic Skills Progress: Basic Math Kit – Second Edition developed by Drs. Lynn & Dough Fuchs, Vanderbilt University. Available through Pro-Ed: http://www.proedinc.com/

Math: Vocabulary

☐ Math Vocabulary Probes (Howell, 2008) ⌛: 5 minutes Administration: Group
Description: Students are given a math vocabulary probe consisting of 20 vocabulary items. There are two versions commonly used: (1) The sheet contains vocabulary terms on one side of the sheet and the definitions of those terms—in scrambled order—on the other. The student connects term to its correct definition; (2) The sheet contains only definitions. The student must read each definition and write the correct corresponding vocabulary term.
Where to get materials: • Math vocabulary probes are developed by the school. Teachers create 'vocabulary pools' that contain the key vocabulary items to be included in probes. From that larger pool, vocabulary items are randomly sampled to create individual probes.

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