Linear Algebra MA233

1. Let T:$\mathbb{R}^5 \to \mathbb{R}^3$ be a MAP THAT WE WOULD LIKE TO BE a linear transformation.
   Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$.

   Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \\ 2 \end{bmatrix}, v_5 = \begin{bmatrix} 2 \\ 1 \\ -2 \\ B \\ 2 \end{bmatrix}$.

   Let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, u_4 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} u_5 = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$

   (a) Show that B can be 0.

   (b) What value(s) can't B be? Give a valid reason.

2. Let T:$\mathbb{R}^5 \to \mathbb{R}^3$ be a linear transformation.
   Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$.

   Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \\ 2 \end{bmatrix}, v_5 = \begin{bmatrix} 2 \\ 1 \\ -2 \\ 0 \\ 2 \end{bmatrix}$.

   Let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, u_4 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} u_5 = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$

   (a) What is the matrix representation of T. You need to compute an inverse of a matrix.

(b) Give the matrix representation of T AS THE PRODUCT OF TWO MATRICES.

(c) What it T(2,5,0,-4,7)?

(d) Give a basis for the null space of T?

(e) Give a basis for the column space of T.

3. Let T:$(\mathbb{Z}_3)^5 \to (\mathbb{Z}_3)^5$ be a MAP THAT WE WOULD LIKE TO BE a linear transformation.
Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$.

Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \\ 2 \end{bmatrix}$, $v_5 = \begin{bmatrix} 2 \\ 1 \\ -2 \\ B \\ 2 \end{bmatrix}$.

Let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $u_4 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ $u_5 = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$

(a) Show that B can be 0.

(b) What value(s) can't B be? Give a valid reason.

4. Let T:$(\mathbb{Z}_3)^5 \to (\mathbb{Z}_3)^5$ be a linear transformation.
Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$.

Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $v_4 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, $v_5 = \begin{bmatrix} 2 \\ 1 \\ -2 \\ 0 \\ 2 \end{bmatrix}$.

Let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $u_4 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ $u_5 = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$

(a) What is the matrix representation of T. Give the matrix representation of the AS THE PRODUCT OF TWO MATRICES.

(b) What it T(2,1,0,1,1)?

(c) Give a basis for null space of T.

(d) Give a basis for the column space of T (range of T).

(e) Give the entire column space (range of T).