Linear Algebra MA233

1. Let VS be a vector space (over $\mathbb{R}$) and suppose v_1, v_2, v_3, v_4, v_5 are vectors in VS. Know defintition of all vector space concepts, defintions and axioms and be able to give defintions and answer true false questions with supportive reasons.

2. Let T:$\mathbb{R}^6 \to \mathbb{R}^3$ be a linear transformation. Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$, $T(v_6) = u_6$. Prove that if
$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, then $\{v_1, v_2, v_3\}$ is independent.

3. Let T:$\mathbb{R}^6 \to \mathbb{R}^4$ be a linear transformation. Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$, $T(v_6) = u_6$. Prove that if
$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, u_4 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, then the null space of T must have dimension at least 3.

4. Let T:$\mathbb{R}^6 \to \mathbb{R}^3$ be a linear transformation. Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$, $T(v_6) = u_6$. Prove or provide counter example:

If v_1, v_2, v_3 are independent, then u_1, u_2, u_3 are independent.

5. Let T:$\mathbb{R}^3 \to \mathbb{R}^5$ be a linear transformation. Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$ Prove that if

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ and } u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ 5 \end{bmatrix} \text{ then T is one-to-one.}$$

6. Let T:$\mathbb{R}^5 \to \mathbb{R}^3$ be a linear transformation. Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$.

Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, v_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$

Let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}, u_4 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$

(a) What is the matrix representation of T?

(b) What it T(2,3,0,-2,7)?

(c) Give a basis for the null space of T.

(d) Give a basis for the column space (range) of T.

7. Let T:$(\mathbb{Z}_3)^5 \to (\mathbb{Z}_3)^5$ be a linear transformation. Let $T(v_1) = u_1$, $T(v_2) = u_2$, $T(v_3) = u_3$, $T(v_4) = u_4$, $T(v_5) = u_5$.

Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, v_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

Let $u_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, $u_4 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $u_3 = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}$

(a) What is the matrix representation of T?

(b) What it T(2,1,0,1,1)?

(c) Give a basis for the null space of T.

(d) How many vectors are there in the null space of T?

(e) Give three distinct non zero vectors in the null space and indicate how you found them.

(f) Give a basis for the column space of T (range of T).

(g) Give the entire column space (range of T).