Analyt II Notes

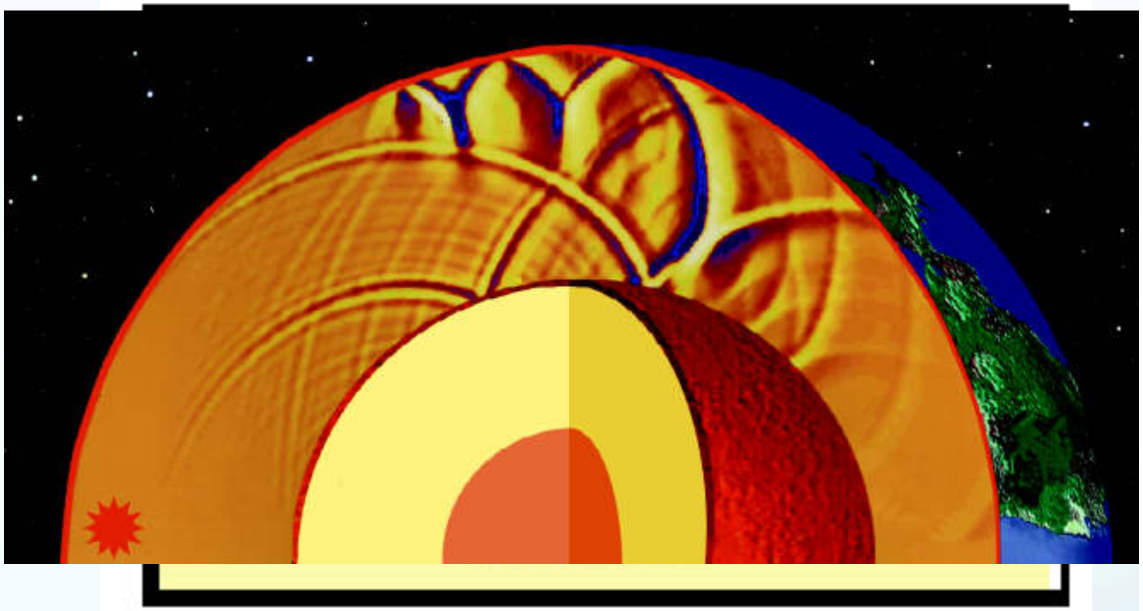
- Homework-Ch15
  - Due on Monday

- We will have Quiz 1 on Monday at 8:30am sharp!
  - Make sure your lockdown browser works
  - use “lockdown browser test” on WebAssign
  - No make-up quiz
  - Covers Ch. 15’s content

Chapter 16: Waves - I

Waves are a primary subject in Physics

- Music and sounds
- Light
- Ripples on water



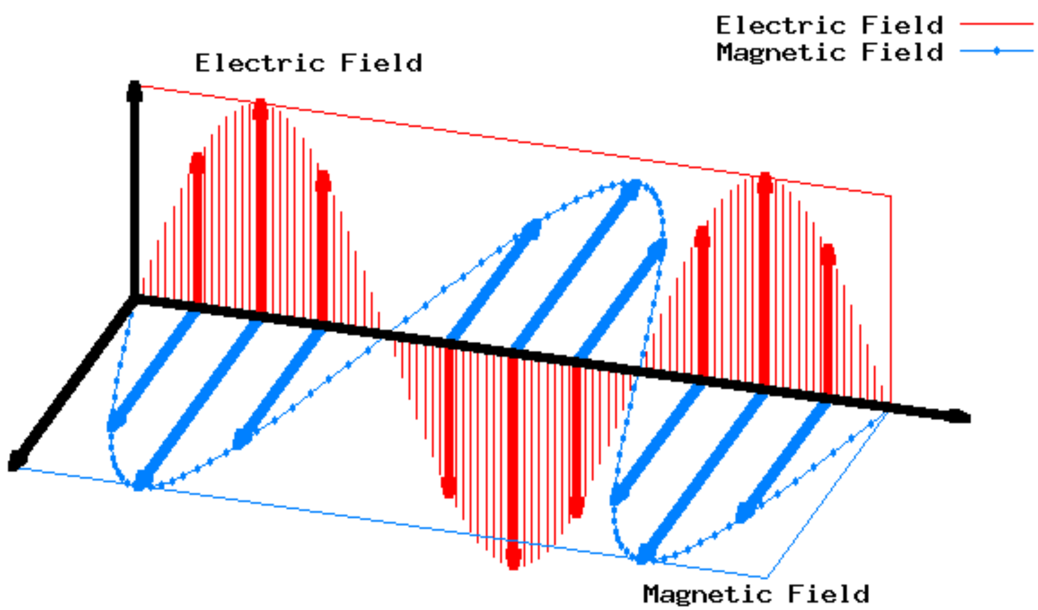
Types of waves :

1. Mechanical waves
   - ➢ Involves disturbance of material
   - Water waves
   - Sound waves
   - Seismic waves
   - ➢ Need a medium to propagate

Chapter 16: Waves - I

Types of waves:

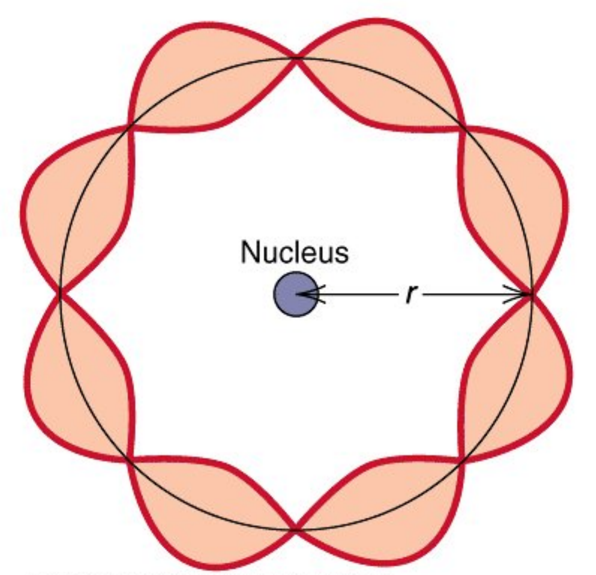
2. Electromagnetic (EM) waves
   - Light (UV, radio, visible, x-rays, etc)
   - Made of electric+magnetic components
   - All EM waves travel at same speed ($c \cong 3\times10^8$ m/s)
   - No medium needed to propagate

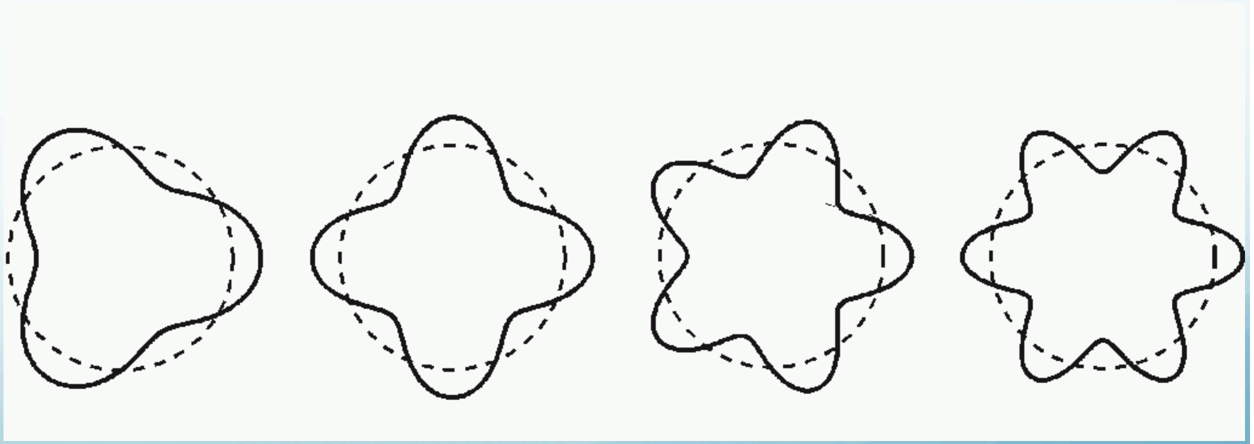


Chapter 16: Waves - I

Types of waves:

3. Matter waves
   - Associated with fundamental particles (protons, electrons)
   - Reflects wave-particle duality
   - It's a concept of quantum mechanics
   - Also called "de Broglie waves"



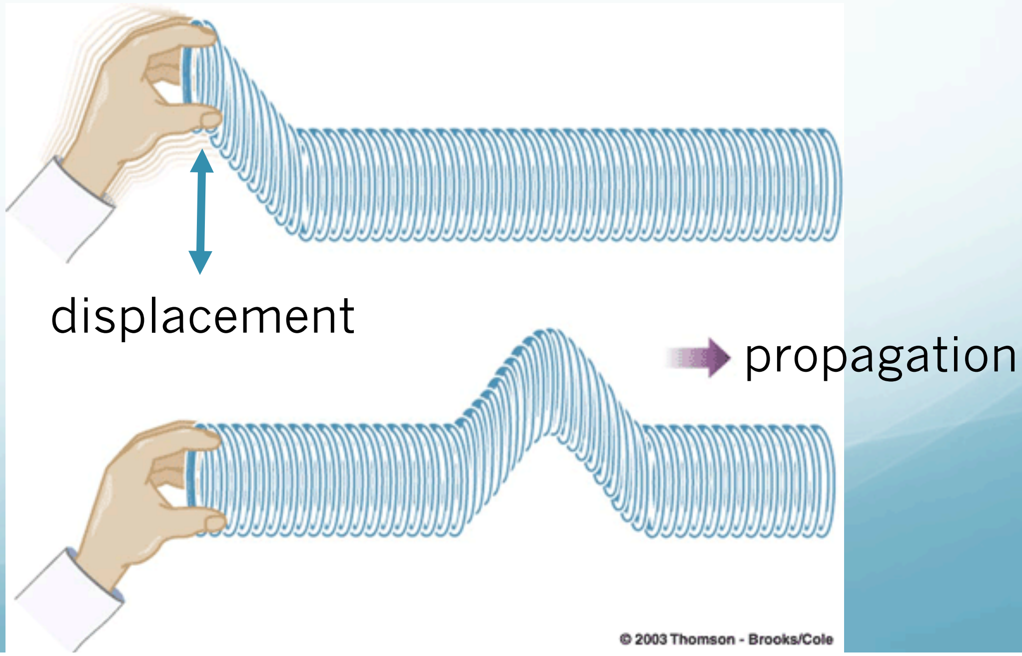


§16-1: Transverse Waves

- A **transverse wave** is
  - disturbance for which the displacement is **perpendicular** to the propagation direction
- The displacement forces the next section to move
  - Generates chain reaction of motion when the displacement changes with time (wave velocity)

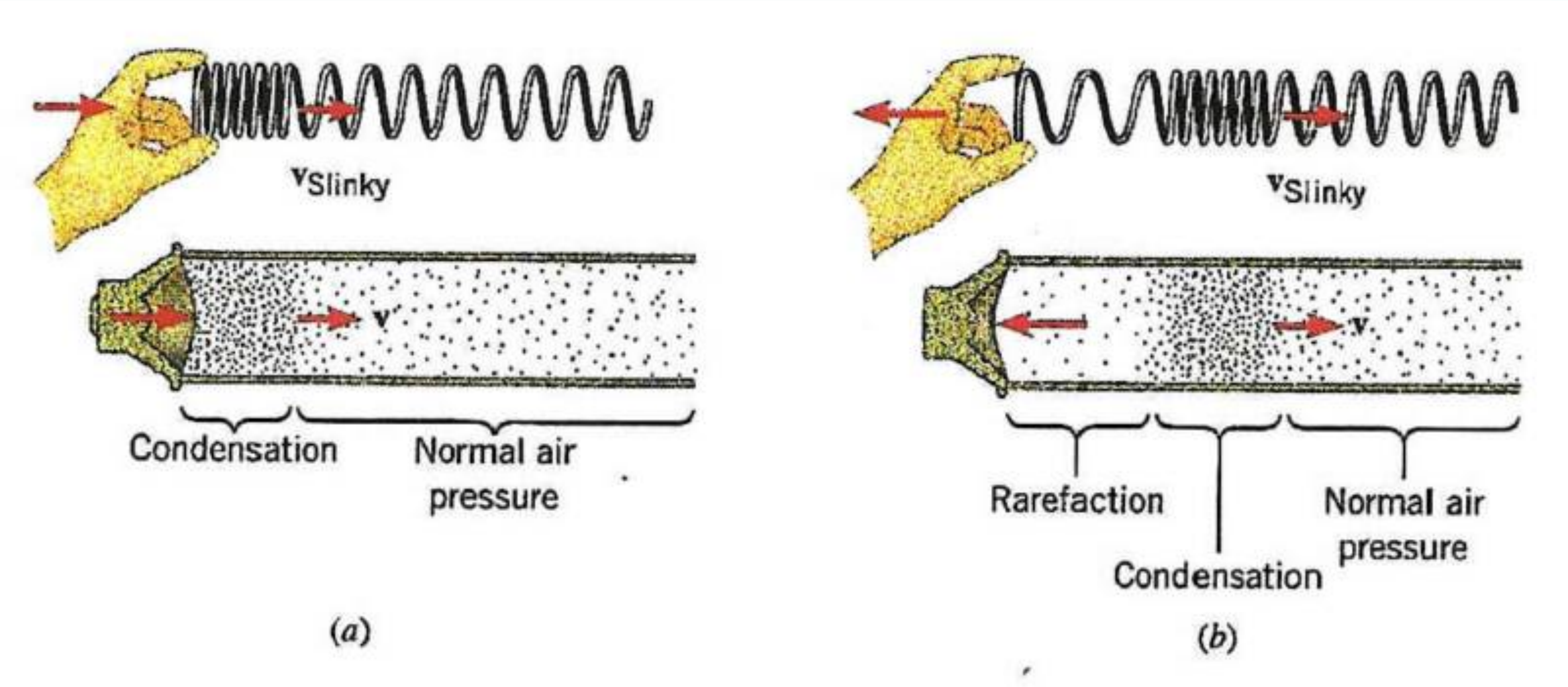
Video examples:

- 2 pulses (animation)
- 1 pulse (real life)

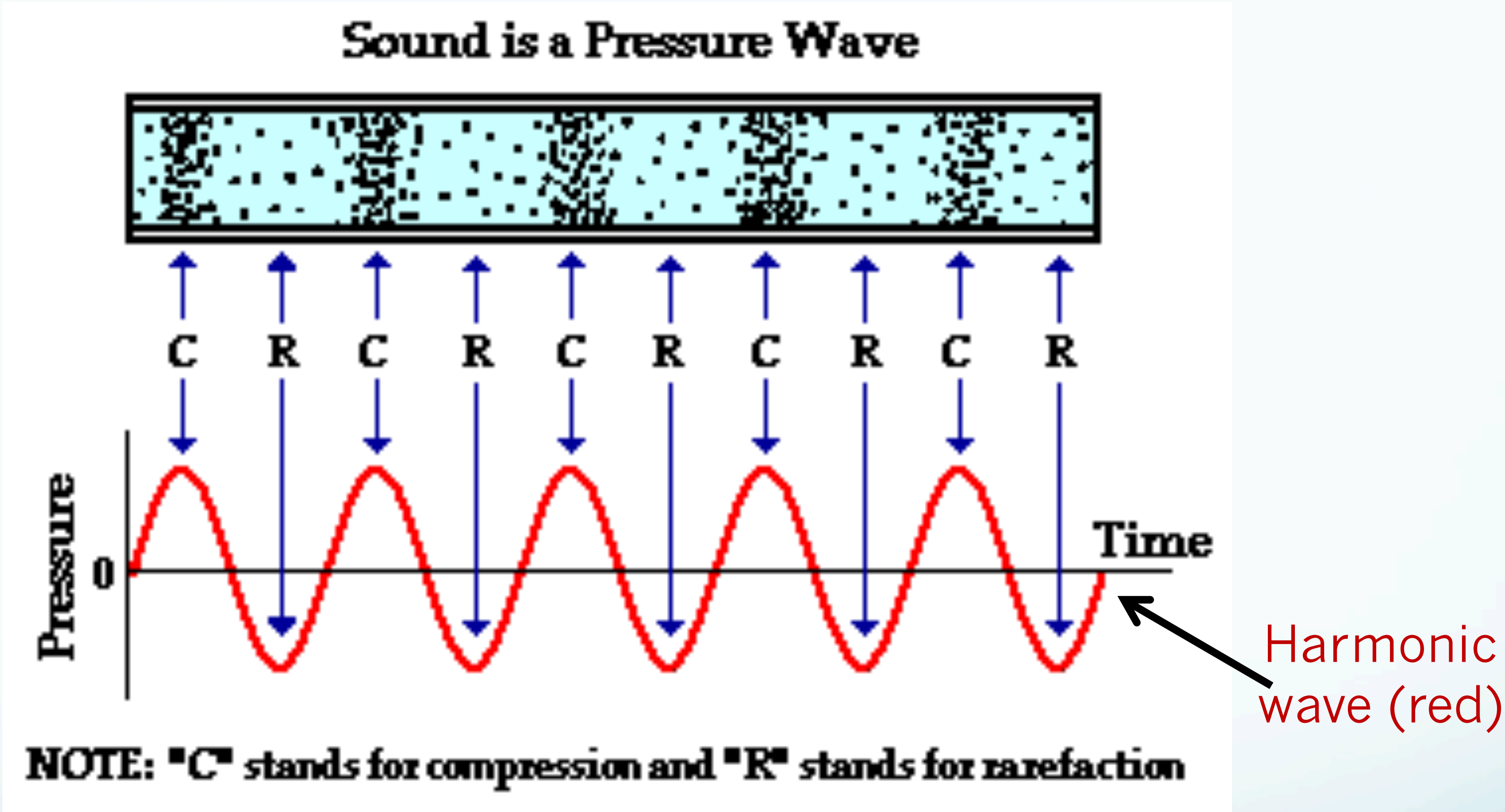


§16-1: Longitudinal Waves

- A **longitudinal** wave is
  - disturbance for which the displacement is **parallel** to the propagation direction
- The displacement forces the material to compress and decompress



Sound Wave is Longitudinal

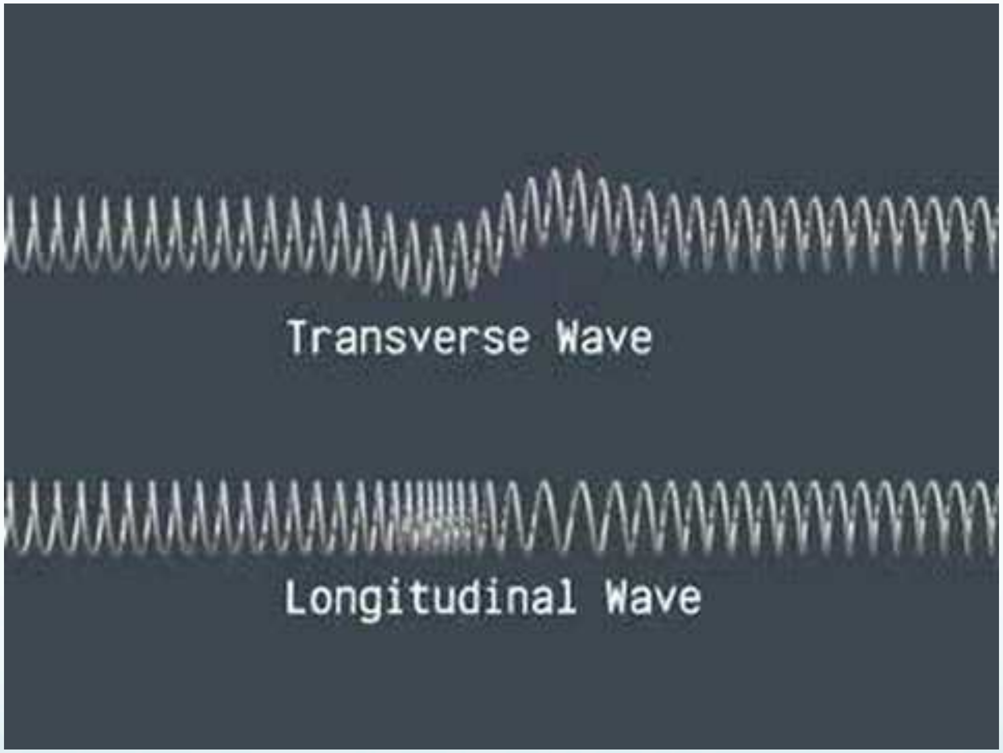


Plotting pressure of air vs time, we can see the wave

(that why it is also described as a compression wave)

Transversal VS Longitudinal

- Difference is the direction of displacement with respect to the direction of propagation of wave



- Both are **"travelling" waves**, since they move from A to B
- Combination of multiple waves can lead to "standing" waves

Wave Function

How to fully quantify a wave?

→ Need a function with a wave shape

$$y = f(x,t)$$

Where

y is the resulting displacement

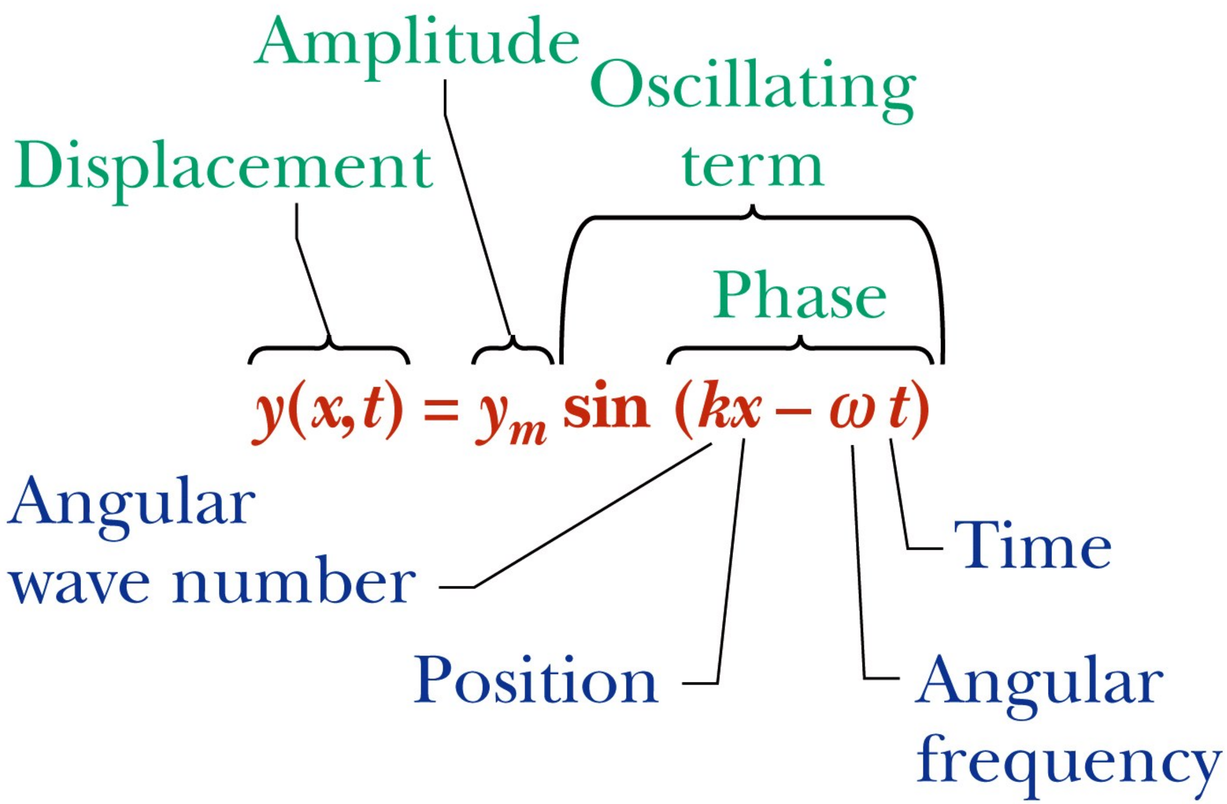
x is along the propagation direction

t is time

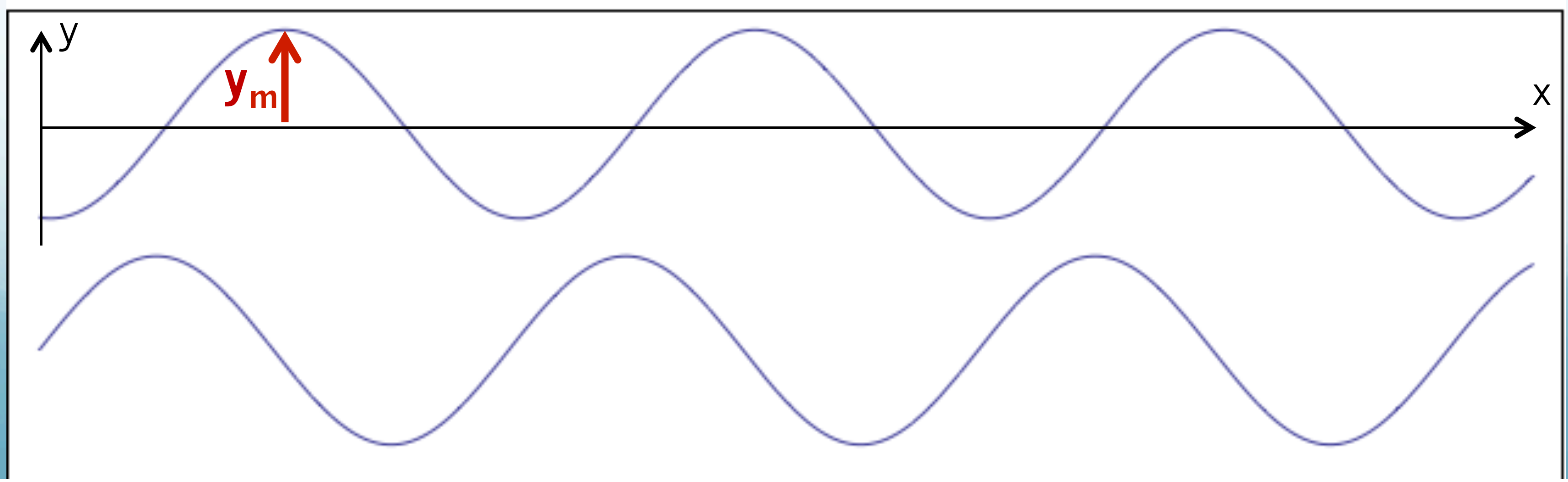
f(x,t) is a function that changes in time and space.

Generally, the function has a sinusoidal shape:

$$\mathbf{y(x,t) = y_{max} \sin (}kx - \omega t\mathbf{)}$$



Amplitude
Oscillating term
Displacement
Phase
$y(x,t) = y_m \sin(kx - \omega t)$
Angular wave number
Time
Position
Angular frequency



y
y_m
x

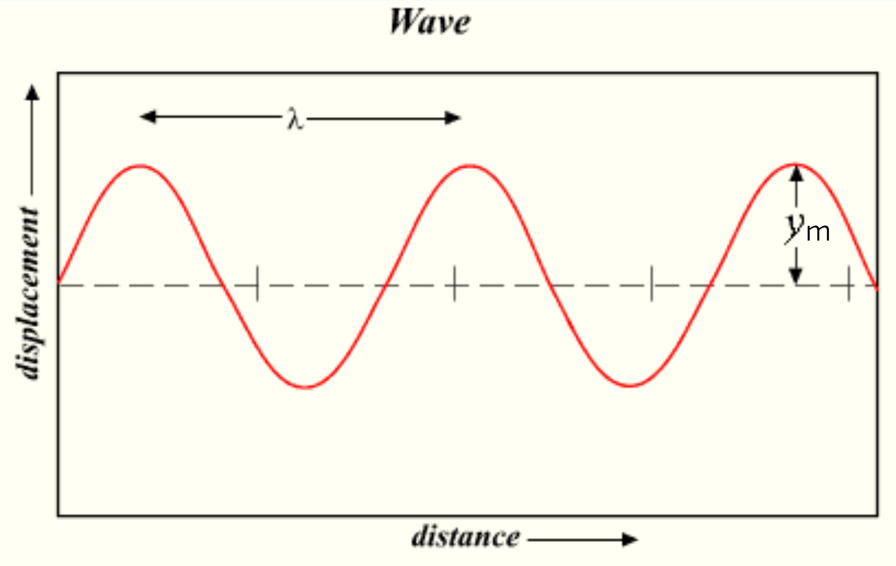
§16-1: Wavelength and Wave Number

Phase:

- It's the argument of sine function, $kx - \omega t$
- Most of the wave information is in the phase

Wavelength λ and Angular Wave Number k:

- λ is the distance between repetitions of the shape of the wave (parallel to propagation direction)



§16-1: Wavelength and Wave Number

λ and k are linked together.

At a given time, say t=0, the wave function is

$$y(x,0) = y_m \sin kx$$

At both ends of the wave, the value of y(x,0) is the same, so

$$y_m \sin kx_1 = y_m \sin k(x_1 + \lambda)$$
$$= y_m \sin(kx_1 + k\lambda)$$

Because a sine function repeats itself every 2π rad, that implies $k\lambda$ = 2π (based on eq above). So...

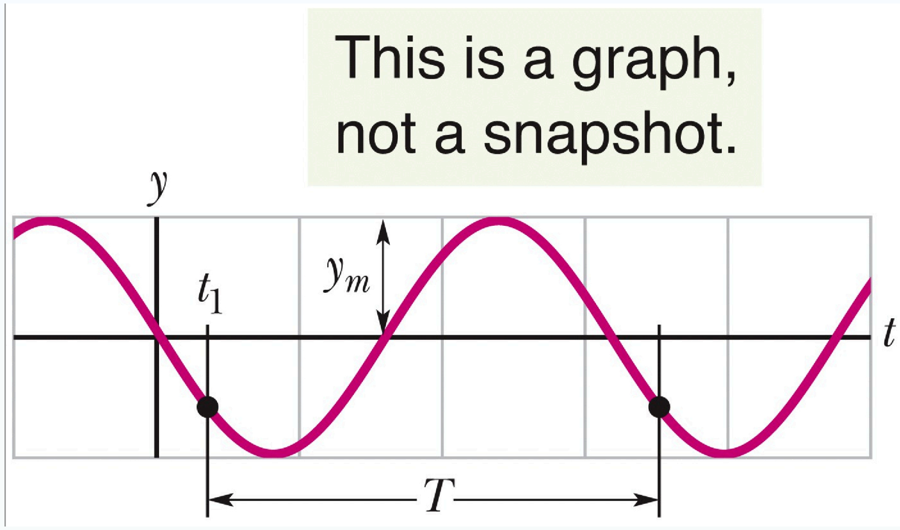
$$k = \frac{2\pi}{\lambda}$$

angular wave number

(Not the spring constant!!!)

§16-1: Angular Frequency and Period

Frequency ω and period *T* are also linked.

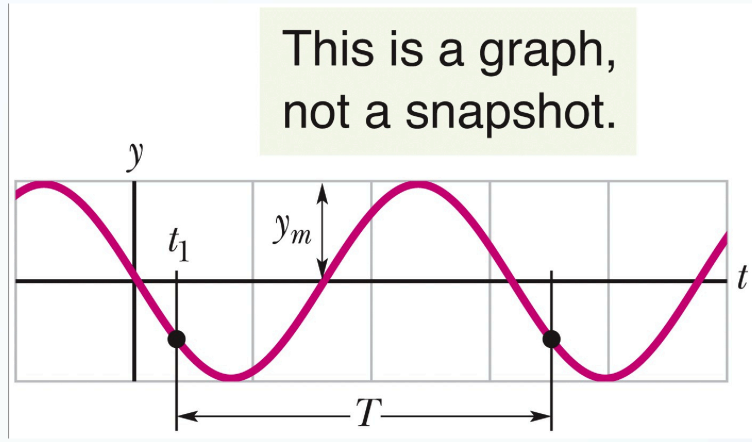


At a given position, say *x*=0, the string element is moving up and down with time:

$$y(0,t) = y_m \sin(-\omega t) \\ = -y_m \sin \omega t \qquad (x = 0)$$

§16-1: Angular Frequency and Period

The time it takes for the wave to move thru a full oscillation, i.e. its period *T*, is the "time" between two identical y values.



Like we did for *λ* and *k* we find that

$$-y_m \sin \omega t_1 = -y_m \sin \omega(t_1 + T) = -y_m \sin(\omega t_1 + \omega T)$$

Therefore, since $\omega T = 2\pi$ rad, then

$$\omega = \frac{2\pi}{T}$$

angular frequency

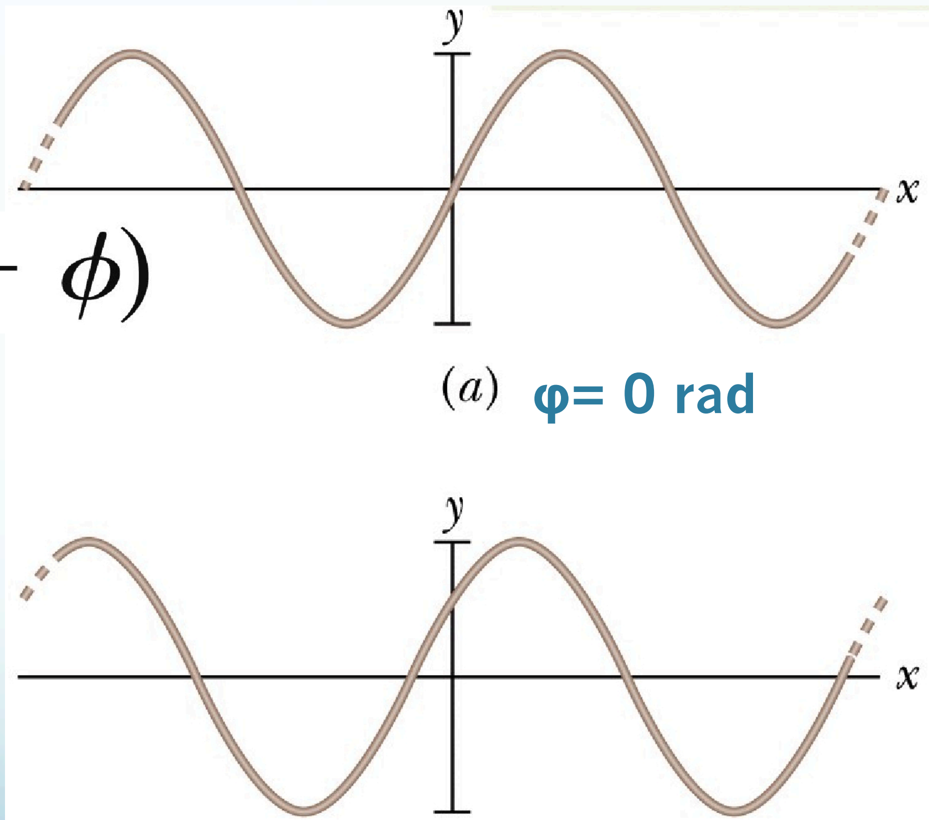
§16-1: Phase constant φ

What if a wave doesn't start at $y(0,0) = 0$?

→Means there is a shift, called **phase constant ϕ**

The wave function becomes:

$$y = y_m \sin(kx - \omega t + \phi)$$



(a) **φ= 0 rad**

(b) **φ= π/5 rad**

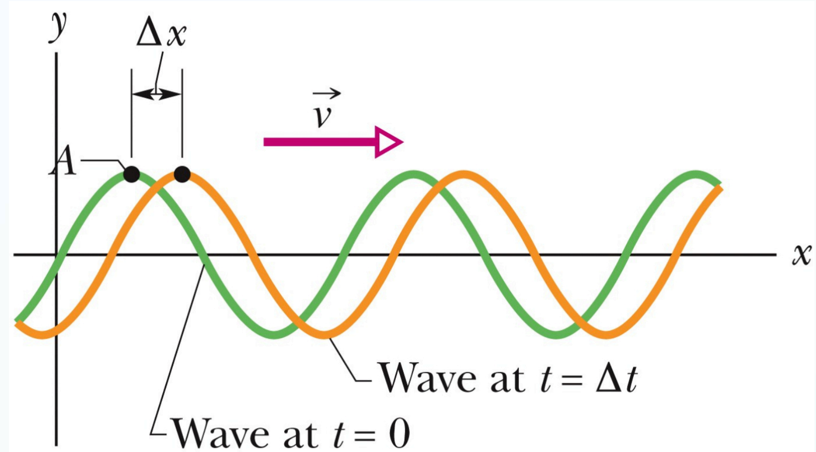
The wave function has

- same y_m value
- same ω value
- *same k* value
- different φ value

§16-1: Speed of Traveling Wave

Two snapshots of a wave are taken a small time interval Δt apart:

- Travelling in +x direction
- Wave speed is $\Delta x/\Delta t$ (or also dx/dt)



Assuming the wave retains its displacement (i.e. Δx doesn't change with for fixed Δt *value*), that means

$$kx - \omega t = \text{a constant}$$

(in other words, only a φ value is needed to shift wave)

We can now derive this expression to obtain dx/dt...

§16-1: Speed of Traveling Wave

$$kx - \omega t = \text{a constant}$$

Deriving with respect to time:

$$\frac{d}{dt}(kx - \omega t) = k\frac{dx}{dt} - \omega = 0$$

Rearranging the terms: $$\frac{dx}{dt} = v = \frac{\omega}{k}$$

Knowing that $k = 2\pi/\lambda$ and $\omega = 2\pi/T$, we obtain the expression for the **wave speed**:

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

Fundamental relation!!!