## What is the charge enclosed?

core: $\rho_{1}$, radius $a$ thin shell: $\sigma$, radius $b$ thick shell: $\rho_{2}$, radii $c \& d$ Gaussian sphere, radius $r$

$$
\frac{4}{3} \pi a^{3} \rho_{1}+4 \pi b^{2} \sigma+\frac{4}{3} \pi\left(r^{3}-c^{3}\right) \rho_{2}
$$

Find $E$ at radius $r$ :

$$
\begin{aligned}
& \varepsilon_{0} 4 \pi r^{2} E=\frac{4}{3} \pi a^{3} \rho_{1}+4 \pi b^{2} \sigma+\frac{4}{3} \pi\left(r^{3}-c^{3}\right) \rho_{2} \\
& E=\frac{1}{3 \varepsilon_{0} r^{2}}\left(a^{3} \rho_{1}+3 b^{2} \sigma+\left(r^{3}-c^{3}\right) \rho_{2}\right)
\end{aligned}
$$

Take a very long insulating rod, radius $a=0.05 \mathrm{~m}$ and charge density $\rho=60 \mu \mathrm{C} / \mathrm{m}^{3}$, surrounded by a conducting hollow cylinder of inner radius $b=0.10 \mathrm{~m}$ and outer radius $c=0.15 \mathrm{~m}$.

What is the magnitude of $E$ at $a<r<b$ ?

$$
\begin{gathered}
\varepsilon_{0} \Phi_{E}=q_{\mathrm{enc}} \\
\varepsilon_{0}\left(2<\not r E_{r}\right)=\rho\left(a^{2} \not h\right) \\
E_{r}=\frac{\rho a^{2}}{2 \varepsilon_{0} r} \\
E_{r}=\frac{\left(60 \mu \mathrm{C} / \mathrm{m}^{3}\right)(0.05 \mathrm{~m})^{2}}{2\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right) r}=\frac{8475 \mathrm{Nm} / \mathrm{C}}{r}
\end{gathered}
$$



Take a very long insulating rod, radius $a=0.05 \mathrm{~m}$ and charge density $\rho=60 \mu \mathrm{C} / \mathrm{m}^{3}$, surrounded by a conducting hollow cylinder of inner radius $b=0.10 \mathrm{~m}$ and outer radius $c=0.15 \mathrm{~m}$.
What is the magnitude of $E$ at $b<r<c$ ?

$$
E=0 \quad!
$$

$$
\varepsilon_{0} \Phi_{E}=q_{\mathrm{enc}}
$$

$$
0=\rho\left(a^{2} \not h\right)+\sigma(2 \pi b h)
$$

$$
\sigma=-\frac{\rho a^{2}}{2 b}
$$



$$
\sigma=-\frac{\left(60 \mu \mathrm{C} / \mathrm{m}^{3}\right)(0.05 \mathrm{~m})^{2}}{2(0.10 \mathrm{~m})}=-0.75 \mu \mathrm{C} / \mathrm{m}^{2}
$$

Same setup: long insulating rod, radius $a=0.05 \mathrm{~m}, \rho=60 \mu \mathrm{C} / \mathrm{m}^{3}$ conducting pipe inner and outer radius $b=0.10 \mathrm{~m}, c=0.15 \mathrm{~m}$, but now with charge $\lambda=15 \mu \mathrm{C} / \mathrm{m}$.
What is the magnitude of $E$ at $r>c$ ?

$$
\begin{gathered}
\varepsilon_{0} \Phi_{E}=q_{\mathrm{enc}} \\
\varepsilon_{0}\left(2 \pi r \not h E_{r}\right)=\rho\left(\pi a^{2} \not h\right)+\lambda \not h \\
E_{r}=\frac{\rho \pi a^{2}+\lambda}{2 \pi \varepsilon_{0} r} \\
E_{r}=\frac{\left(60 \mu \mathrm{C} / \mathrm{m}^{3}\right) \pi(0.05 \mathrm{~m})^{2}+(15 \mu \mathrm{C} / \mathrm{m})}{2 \pi\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}\right) r}=\frac{2.78 \times 10^{5} \mathrm{Nm} / \mathrm{C}}{r}
\end{gathered}
$$

