Analyt I, Equation Sheet, Final Exam

Angular Velocity and Acceleration

$$w = \frac{dq}{dt}$$

$$a = \frac{dw}{dt} = \frac{d^2q}{dt^2}$$

Constant Angular Acceleration

$$w = w_0 + at$$

$$q - q_0 = w_0 t + \frac{1}{2}at^2$$

$$w^2 = w_0^{\ 2} + 2a(q - q_0)$$

Linear and Angular Variables

$$s = q\, r$$
$$v = w\, r$$
$$a = a\, r$$

Moment of Inertia

$$I = \sum_i m_i r_i^2$$

$$I = \int r^2 dm$$

See additional page for more rotational inertias

Rotational Kinetic Energy

$$K = \frac{1}{2} I w^2$$

Torque

$$\vec{t} = \vec{r} \times \vec{F}$$

$$t = rF \sin f = r_\perp F = rF_t$$

Newton's Second Law, Angular Form

$$\sum t = Ia$$

$$\sum \vec{t} = \frac{d\vec{L}}{dt}$$

Work and Power for Rotation

$$W = \int_{q_i}^{q_f} t\, dq$$

$$P = \frac{dW}{dt} = t\, w$$

Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = mvr \sin f = mvr_\perp = mrv_t$$

$$L = Iw$$

Frequency, Period, Angular Frequency

$$f = \frac{1}{T} = \frac{w}{2p}$$

$$w = 2p\ f = \frac{2p}{T}$$

$$T = \frac{1}{f} = \frac{2p}{w}$$

Simple Harmonic Motion

$$x(t) = A\cos(w\,t + f)$$

$$v(t) = -wA\sin(w\,t + f)$$

$$a(t) = -w^2 A\cos(w\,t + f)$$

Spring-Mass system

$$w = \sqrt{\frac{k}{m}} \qquad T = 2p\sqrt{\frac{m}{k}}$$

Total Mechanical Energy in Spring-Mass system

$$E = \frac{1}{2}kA^2$$

Simple Pendulum

$$w = \sqrt{\frac{g}{L}} \qquad T = 2p\sqrt{\frac{L}{g}}$$

Physical Pendulum

$$w = \sqrt{\frac{mgh}{I}} \qquad T = 2p\sqrt{\frac{I}{mgh}}$$

Newton's Law of Gravity

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} Nm^2 / kg^2$$

Gravitational Potential Energy

$$U = -\frac{GMm}{r}$$

Kepler's Third Law

$$T^2 = \left(\frac{4p^2}{GM}\right)a^3$$

Energy in Planetary Motion

$$E = -\frac{GMm}{2a}$$

Newton's 2nd Law

$$\sum \vec{F} = m\vec{a}$$

Friction

$$f_{s,\max} = m_s N$$
$$f_k = m_k N$$

Work and Energy

$$K = \frac{1}{2}mv^2$$
$$W = \vec{F} \cdot \vec{d} = Fd\cos f$$
$$W = \int_{x_i}^{x_f} F(x)dx$$
$$W = \int \vec{F} \cdot d\vec{s}$$
$$W = \Delta K$$

Springs

$$F = -kx$$
$$W_s = \frac{1}{2}kx_i{}^2 - \frac{1}{2}kx_f{}^2$$
$$W_s = -\frac{1}{2}kx^2$$

Power

$$P_{av} = \frac{W}{\Delta t}$$
$$P = \frac{dW}{dt}$$
$$P = \vec{F} \cdot \vec{v}$$

Potential Energy

$$\Delta U = -W$$
$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$

Gravitational Potential Energy

$$U = mgh$$

Elastic Potential Energy

$$U = \frac{1}{2}kx^2$$

Potential Energy Curves

$$F(x) = -\frac{dU(x)}{dx}$$

Generalized Work-Energy Theorem

$$W_{NC} = \Delta E$$

Linear Momentum

$$\vec{p} = m\vec{v}$$

Rocket Equations

$$Thrust = Ru$$
$$v_f - v_i = u\ln\left(\frac{M_i}{M_f}\right)$$

Impulse

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)dt$$
$$\vec{J} = \Delta\vec{p}$$
$$\vec{F}_{av} = \frac{\vec{J}}{\Delta t}$$

Center of Mass

$$x_{cm} = \frac{1}{M}\sum_i m_i x_i$$
$$y_{cm} = \frac{1}{M}\sum_i m_i y_i$$
$$z_{cm} = \frac{1}{M}\sum_i m_i z_i$$

$$x_{cm} = \frac{1}{M}\int x\,dm$$
$$y_{cm} = \frac{1}{M}\int y\,dm$$
$$z_{cm} = \frac{1}{M}\int z\,dm$$

$$\vec{r}_{cm} = \frac{1}{M}\sum_i m_i \vec{r}_i$$

1D Elastic collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$$
$$v_{1f} = \frac{(m_1 - m_2)v_{1i} + 2m_2 v_{2i}}{m_1 + m_2}$$
$$v_{2f} = \frac{2m_1 v_{1i} - (m_1 - m_2)v_{2i}}{m_1 + m_2}$$

1D Displacement, Speed, Velocity, Acceleration

$$\Delta x = x_2 - x_1$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$s_{av} = \frac{total\,distance}{\Delta t}$$

$$v = \frac{dx}{dt}$$

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

Constant Acceleration

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = {v_0}^2 + 2a(x - x_0)$$

Free-Fall Motion

$$v = v_0 - gt$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

$$v^2 = {v_0}^2 - 2g(y - y_0)$$

2D Motion

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k} = \vec{r}_2 - \vec{r}_1$$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$v_x = \frac{dx}{dt},\; v_y = \frac{dy}{dt},\; v_z = \frac{dz}{dt}$$

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$a_x = \frac{dv_x}{dt},\; a_y = \frac{dv_y}{dt},\; a_z = \frac{dv_z}{dt}$$

Projectile Motion

$$x - x_0 = (v_0 \cos q)\, t$$

$$y - y_0 = (v_0 \sin q)\, t - \frac{1}{2} g t^2$$

$$v_y = v_0 \sin q - gt$$

$${v_y}^2 = (v_0 \sin q)^2 - 2g\Delta y$$

$$\Delta y = (\tan q)\Delta x - \frac{g(\Delta x)^2}{2(v_0 \cos q)^2}$$

Vectors

$$\vec{a}\cdot\vec{b} = ab\cos f = a_x b_x + a_y b_y + a_z b_z$$

$$\left|\vec{a}\times\vec{b}\right| = ab\sin f$$

$$\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

$$a = |\vec{a}| = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$$

Uniform Circular Motion

$$a = \frac{v^2}{r}$$

$$T = \frac{2p\,r}{v}$$

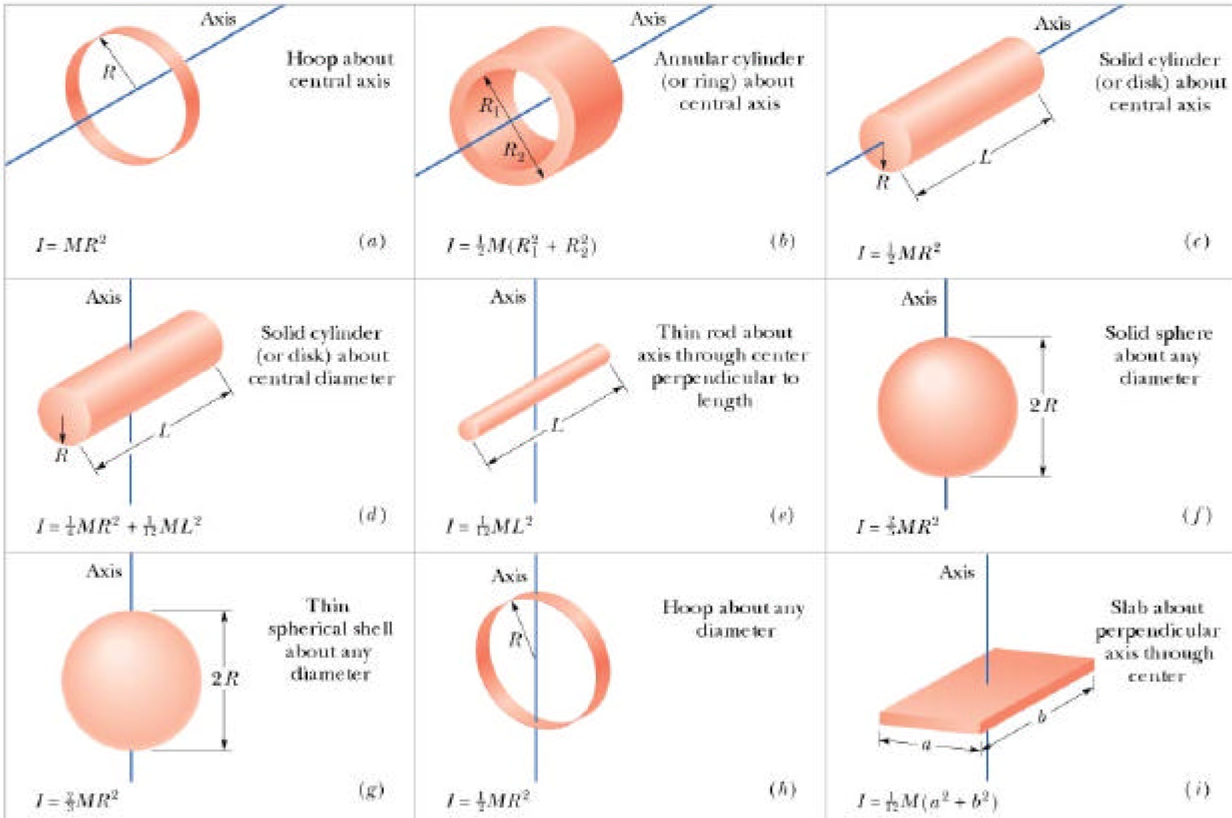
$$f = \frac{1}{T} = \frac{v}{2p\,r}$$

Relative Velocity

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos q$$


Axis
Hoop about central axis
$I = MR^2$
(a)
Axis
Annular cylinder (or ring) about central axis
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(b)
Axis
Solid cylinder (or disk) about central axis
$I = \frac{1}{2}MR^2$
(c)
Axis
Solid cylinder (or disk) about central diameter
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
$I = \frac{1}{12}ML^2$
(e)
Axis
Solid sphere about any diameter
2R
$I = \frac{2}{5}MR^2$
(f)
Axis
Thin spherical shell about any diameter
2R
$I = \frac{2}{3}MR^2$
(g)
Axis
Hoop about any diameter
$I = \frac{1}{2}MR^2$
(h)
Axis
Slab about perpendicular axis through center
$I = \frac{1}{12}M(a^2 + b^2)$
(i)