(Object) Integration in Physics

This type of integral applies when you have an extended charged object that contributes to some physical quantity (e.g., total charge, electric potential, electric field), but different parts of the object contribute in different ways. For example, maybe you need the electric field $\vec{E}$.

> The general method is **PACCEI**: Chop the object in **Pieces**, find **Axes** to describe the chopping, find the **Contribution** from each small piece, determine the vector **Component** (if appropriate), **Express** that contribution in terms of the coordinates, and **Integrate**.

To keep track of things: any quantity that is really small should be represented by a differential (e.g. *dq* for a small charge). Note that this is **not** the product of *d* and *q*. Rather, it is a two-letter variable for a single quantity.

A) Chop the object into small **PIECES**. All pieces must have the same sort of shape, and you must already know a formula (e.g., for $\vec{E}$) for this shape. For rope-like objects, the pieces will always be point charges. We'll call the small charge on each small piece *dq*. **Clearly mark this *dq* on your diagram.**

B) Choose good **AXES**. One coordinate must be a way to describe the positions of the different small pieces. This might be a straight axis (*x* or *y*), a circular arc (θ), or a distance from a point (*r*). The length or width of one small piece will then be called *dx*, *dy*, *dr*, or arc length $ds = R\,d\theta$. For potential & E-field, the problem will also involve a point of interest; try to have an axis passing through this point.

C) Find an expression giving the **CONTRIBUTION** of one small piece to the overall physical quantity. In doing so, you will consider the small piece as a shape for which you know the formula. For instance, you might consider your object to be composed of many small pieces that are effectively point charges. This contribution will be a small part of the total (e.g., $dQ, dV, d\vec{E}$). It should be proportional to the small size of one piece.

   It is important that this expression work for **any** piece, not just one at a special location. Use your coordinates (from part B) to describe the position of this arbitrary piece.

D) If the physical quantity you are trying to get is a vector, find one **COMPONENT** of the CONTRIBUTION. If you need more than one vector component, finish these instructions for one, then come back to step (D) for the next component.

E) **EXPRESS** the contribution in terms of only **constants** and the variables of your **coordinate system**. This may involve relating one small quantity to another (e.g., a small charge to a small length $dq = \lambda\, dx$, or a small angle to a small length $ds = R\,d\theta$).

F) **INTEGRATE** your expression over the charge distribution. That is, put an integral sign in front of your expression, then use your favorite method to evaluate the integral. The limits of the integral should describe the edges of your object.

G) Publish results to world media. Gain fame and fortune.