

## Types of Gauss' Law Problems

There are a limited number of Gauss' Law problems, and certain types of equations that will always come up in each type.

Notation: Below, I'll always use  $r$  for the radius of the Gaussian surface, and use  $a$  for the radius of physical charged objects. It is rarely the case that you want  $r$  and  $a$  to be the same.

Problem Type	Charge densities likely to be involved
<p><b>Conductor with Cavity</b>  <i>Always</i> use a Gaussian surface that passes through “the meat” of the conductor (shape is unimportant), so that</p> $0 = \epsilon_0 \Phi = q_{\text{enc}} = Q_{\text{surrounded}} + Q_{\text{inner surface}}$ <p><i>Likely</i> to want to use charge conservation on the conductor:</p> $Q_{\text{conductor}} = Q_{\text{outer surface}} + Q_{\text{inner surface}}$	$Q, \sigma$
<p><b>Spherical Symmetry</b>  <i>Always</i> use Gaussian surface (sphere with radius <math>r</math>), so that</p> $\epsilon_0 E (4\pi r^2) = \epsilon_0 \Phi = q_{\text{enc}}$ <p><i>Likely</i> to want to use some combination of the following to get <math>q_{\text{enc}}</math></p> $q = \sigma (4\pi a^2) \quad q = \rho \left( \frac{4}{3} \pi a^3 \right)$	$Q, \sigma, \rho$
<p><b>Cylindrical Symmetry</b> (infinitely long)  <i>Always</i> use Gaussian surface (cylinder with radius <math>r</math> and length <math>h</math>), so that</p> $\epsilon_0 E (2\pi r h) = \epsilon_0 \Phi = q_{\text{enc}}$ <p>and you will <i>always</i> find that <math>q_{\text{enc}} \propto h</math>.</p> <p><i>Likely</i> to want to use some combination of the following to get <math>q_{\text{enc}}</math></p> $q = \lambda h \quad q = \sigma (2\pi a h) \quad q = \rho (\pi a^2 h)$	$\lambda, \sigma, \rho$
<p><b>Planar Symmetry</b> (infinite in two directions)  <i>Always</i> use Gaussian surface (right prism with length <math>L</math> and ends with area <math>A</math>), so that</p> $\epsilon_0 (E_{\text{right end}} A - E_{\text{left end}} A) = \epsilon_0 \Phi = q_{\text{enc}}$ <p>and you will <i>always</i> find that <math>q_{\text{enc}} \propto A</math>.</p> <p>Note that <math>E_{\text{right end}}</math> and <math>E_{\text{left end}}</math> might be positive (field pointing right) or negative (field pointing left).</p> <p><i>Likely</i> to want to use some combination of the following to get <math>q_{\text{enc}}</math></p> $q = \sigma A \quad q = \rho a A$	$\sigma, \rho$
<p><b>Conductor Surface</b>  <i>Always</i> use same Gaussian surface as planar symmetry, one end in conductor, so that</p> $\epsilon_0 (E_{\text{outside}} A) = \epsilon_0 \Phi = q_{\text{enc}} = \sigma A$	$\sigma$