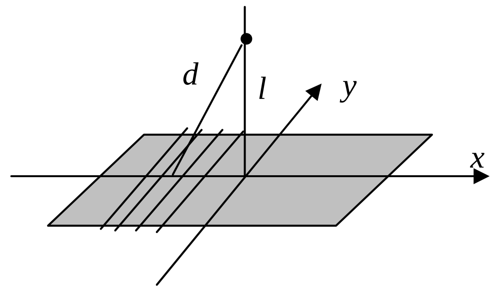
Potential on axis of square

The Problem:



Given a r surface with charge density σ and edge length $2L$, find the electric potential at a point a distance z perpendicularly away from the center of the square.

A Solution:

Chop & Coordinates
Slice plate into many thin rods of length $2L$.
Choose x and y axes in the plane of the square and centered on it, with the y axis parallel to the thin rods. That way, the thin rods have width dx. The z axis passes through the point of interest.

Contribution
An arbitrary rod contributes a potential of

$V = k\lambda \ln\left(\frac{\sqrt{L^2+d^2}+L}{\sqrt{L^2+d^2}-L}\right)$, where d is the distance perpendicularly from the midpoint of the rod to the point of interest. In this case, d is slanted, connecting a point on the x axis with the point of interest on the z axis. We can modify that equation to show that the rod is very thin, and thus contains a very small charge:

$$dV = k\frac{dq}{2L}\ln\left(\frac{\sqrt{L^2+d^2}+L}{\sqrt{L^2+d^2}-L}\right)$$

Express
The charge on one thin rod is $dq = \sigma(\text{area}) = \sigma 2L\,dx$.
The distance d varies depending on which thin rod you consider, and so has to be expressed as $d = \sqrt{z^2+x^2}$. Plugging those in gives:

$$dV = k\sigma \ln\left(\frac{\sqrt{L^2+z^2+x^2}+L}{\sqrt{L^2+z^2+x^2}-L}\right)dx$$

Integrate

$$V = \int_{\text{object}} dV = \int_{-L}^{L} k\sigma \ln\left(\frac{\sqrt{L^2+z^2+x^2}+L}{\sqrt{L^2+z^2+x^2}-L}\right)dx$$

Yeah, right…
But the important thing for this class is that we can get to this integral.