Analytical Physics I, Fall 2004, Final Exam Dr. Pogo

Last name:_______________________________

Displacement, Speed, Velocity, and Acceleration

$?x = x_2 - x_1$

$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$

$\bar{s} = \frac{\text{total distance}}{\Delta t}$

$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

$\bar{a}_x = \frac{\Delta v_x}{\Delta t}$

$a_x = \frac{dv_x}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{d^2 x}{dt^2}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

$\vec{v} = \frac{d\vec{r}}{dt}$

Projectile Motion

(assumes + y is upwards)

$\tan q_0 = \frac{v_{0y}}{v_{0x}}$

$v_{0x} = |v_0| \cos q_0$

$v_{0y} = |v_0| \sin q_0$

$(y - y_0) = \left(\frac{v_{0y}}{v_{0x}}\right)(x - x_0) - \frac{g(x - x_0)^2}{2v_{0x}^2}$

$R = \frac{v_0^2}{g}\sin(2q_0)$

Rockets

Thrust = $Rv_{rel-exhaust}$

$V_f = V_i + v_{rel-exhaust} \ln\left(\frac{m_i}{m_f}\right)$

Center of Mass, Momentum, Impulse

$x_{CM} = \frac{1}{m_{TOT}} \sum_{i=1}^{n} m_i x_i = \frac{1}{m_{TOT}} \int x\,dm$

$x_{CM} = \frac{1}{V_{TOT}} \sum_{i=1}^{n} V_i x_i$ if density is constant

$x_{CM} = \frac{1}{A_{TOT}} \sum_{i=1}^{n} A_i x_i$ for 2D objects

$\vec{v}_{CM} = \frac{1}{m_{TOT}} \sum_{i=1}^{n} m_i \vec{v}_i$

$\vec{v}_{CM}$ = constant, if no external forces act on the system

$\vec{p} = m\vec{v}$

$\sum \vec{F}_{ext} = \frac{d\vec{p}}{dt}$

$\left(\sum \vec{F}_{ave,ext}\right)\Delta t = \Delta\vec{p}$

$\vec{p}_f = \vec{p}_i$ if $\sum \vec{F}_{ext} = 0$

$\vec{J} = \int_{t_1}^{t_2} \vec{F}(t)\,dt$

$\vec{J}_{ave} = \left(\sum \vec{F}_{ave,ext}\right)\Delta t$

Density

$r = \frac{m}{V}$

One Dimensional Elastic Collisions

$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$

$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$

Derivatives

If a and m are constants, and f and g are functions of t:

$\frac{da}{dt} = 0 \qquad \frac{d(f+g)}{dt} = \frac{df}{dt} + \frac{dg}{dt}$

$\frac{daf}{dt} = a\frac{df}{dt} \qquad \frac{d(t^m)}{dt} = mt^{m-1} \quad (m \neq 0)$

Relative Motion

$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$

$\vec{x}_{PA} = \vec{x}_{PB} + \vec{x}_{BA}$

$\vec{a}_{BA} = 0$ (A,B are "inertial")

$\vec{a}_{PA} = \vec{a}_{PB}$ (A,B are "inertial")

Gravity

$g = +9.8\ \text{m/s}^2$

$F = G\frac{m_1 m_2}{r^2}$

$G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

$E_{\text{orbit}} = -\frac{GMm}{2a}$

$U(r) = -\frac{GMm}{r}$

$T = 2p\sqrt{\frac{a^3}{GM}}$

Rotation, Moment of Inertia

$\Delta s = r\Delta q \qquad |v| = |rw| \qquad |a_T| = |ra|$

$w = \frac{dq}{dt} \qquad a = \frac{dw}{dt} \qquad |a_C| = \frac{v^2}{r} = w^2 r$

1 revolution = 360° = 2p radians

constant a:

$q = q_o + w_o t + \frac{1}{2}at^2$

$w = w_o + at$

$w^2 = w_o^2 + 2a(q - q_o)$

$q = q_o + \frac{1}{2}(w + w_o)t$

$q = q_o + wt - \frac{1}{2}at^2$

$T = \frac{2pr}{v} = \frac{2p}{w} = \frac{1}{f}$

$I = \sum m_i r_i^2 = \int r^2 dm = \frac{r}{V_{TOT}} \int r^2 dV = \frac{r}{A_{TOT}} \int r^2 dA$

$I = I_{CM} + mh^2$

Integration

$\int_a^b dx = b - a \qquad \int_a^b c f(x)dx = c\int_a^b f(x)dx$

$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

$\int_a^b x^m dx = \left[\frac{x^{m+1}}{m+1}\right]_{x=a}^{x=b} = \frac{b^{m+1}}{m+1} - \frac{a^{m+1}}{m+1}$ (provided $m \neq$ -1)

Work, Power

$W_F = \vec{F} \cdot \vec{d}$ (constant $\vec{F}$)

$W_F = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$

$W_{\Sigma F} = K_f - K_0$

$K = \frac{1}{2}mV^2$

$\bar{P} = \frac{W}{\Delta t}$

$P = \frac{dW}{dt} = \vec{F} \cdot \vec{V}$

Torque and Angular Momentum

$\vec{t} = \vec{r} \times \vec{F}$

$|\vec{t}| = |r_\perp F|$

$\Sigma\vec{t} = I\vec{a} = d\vec{l}/dt$

$\vec{L} = \Sigma\vec{l} = I\vec{w}$

$\vec{L}_{\text{particle}} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$

$\vec{L}_i = \vec{L}_f$ if $\Sigma\vec{t} = 0$

Vectors

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos f = A_x B_x + A_y B_y + A_z B_z$

$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin f$

$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Potential Energy, Energy Conservation

$U_{F_x 2} - U_{F_x 1} = -\int_{x_1}^{x_2} F_x(x)dx$

$F(x) = -\frac{dU(x)}{dx}$

$U_{2g} - U_{1g} = mg(y_2 - y_1)$ if + y is upwards

$U_s = \frac{1}{2}k\Delta L^2$ assumes $U = 0$ when $\Delta L = 0$

$E = K + U_s + U_g$

$E_2 = E_1$ if all forces are conservative

$E_2 = E_1 + W_{1\to 2, Non-Conservative}$

Rolling

$v_{CM} = Rw$

$K_{ROLLING} = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I_{CM}w^2$

Forces

$\sum \vec{F}_x = m\vec{a}_x$

$\vec{W} = m\vec{g}$

$f_{s,\max} = m_s N$

$f_k = m_k N$

Simple Harmonic Motion

$T = 1/f = 2p/w$

$w = 2pf = \sqrt{k/m}$

$w = \sqrt{k_{\text{effective}}/m}$

$T = 2p\sqrt{L/g}$ (pendulum)

$T = 2p\sqrt{\frac{I}{mgh}}$ (physical pendulum)

$x(t) = x_M \cos(wt + f)$

$v(t) = -wx_M \sin(wt + f)$

$a(t) = -w^2 x_M \cos(wt + f)$

Constant Acceleration

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$v_x = v_{0x} + a_x t$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$x = x_0 + \frac{1}{2}(v_{0x} + v_x)t$

$x = x_0 + v_x t - \frac{1}{2}a_x t^2$

Springs

$|\boldsymbol{F}s| = k\Delta L$

$\boldsymbol{F}_s = -k\boldsymbol{x}$ (if the origin for x is at the undisturbed free end of the spring)

$W_s = ½k(x_2^2 - x_1^2)$