Final Exam
(Three hours)

Name: ________________________________

Instructions for the exam:
1) The exam is closed book and closed notes. You may use your calculators. Also, formulae (which may or may not be useful) are provided on the last page of the exam.
2) Read each problem carefully. To receive full credit, you must show all of your work. Include coordinate systems, free body diagrams, equations, units, and pictures as needed. Put a box around your answers. You are not required to solve the problems in order, so if you get stuck, move on to another problem for a while.
3) There are eight problems, each with several parts. The point value for each part is indicated in [square brackets].
4) Print your last name on every page.
5) Unless otherwise stated, you may neglect air resistance and the effects of planetary curvature.
Problem #1: [26 points total] Short answers:

a. [2] Marcus is lost in Cairo. His position as a function of time is given by
   \[ x(t) = (3\text{cm/s}^4)t^4 - (5\text{cm/s}^2)t^2 + 20\text{cm}. \]
   What is his acceleration when \( t = 2\text{s} \)?

b. [2] The ark is on a truck moving with velocity \( \vec{V} = -10\hat{i} + 6\hat{j} \text{ km/hr} \). Indy is on a horse
   with velocity \( \vec{V} = -6\hat{i} - 3\hat{j} \text{ km/hr} \). What is Indy’s velocity relative to the truck?

c. Marion is on a ship heading 30° west of north at 4 m/s. If \(+y\) is north, and \(+x\) is east (as usual), what is Marion’s velocity in \( i \cdot j \cdot k \) format?

d. [2] While showing off, a swordsman spins his sword in a perfect circle 5 times in 5 seconds. The sword is 1m long.
   What is the magnitude of the average velocity of the tip of the sword during this time?

\[ 1\text{m} \]

e. [2] The same swordsman then spins his sword in a perfect circle 1.5 times in 1.5 seconds.
   What is the magnitude of the average velocity of the tip of the sword during this time?

f. [2] What is the speed of the tip of the sword in part e)?

g. [2] What is the acceleration of the tip of the sword in part e)?
h. [2] Indiana Jones slides down a gravelly hill of height $h = 10\text{m}$ and $\theta = 60^\circ$ at constant speed. What is the coefficient of kinetic friction between Indy and the hill?

\[ h \]

i. [2] Indy’s whip is coiled into a perfect hoop of radius $R = 0.3\text{m}$, and is hanging on a hook on the wall. When the blows a little, it swings back and forth. What is the period of this oscillation?

\[ R \]

j. [2] If the mass of the whip is $1\text{kg}$, what is the effective spring constant for question 1i)?

k. [2] A plot of Sallah’s position vs. time is shown. List all the regions (e.g., “B-C”) where his acceleration is positive or negative.

Positive: __________________
Negative: __________________

l. [2] A plot of Henry Jones’ position vs. time is shown. Rank the regions (e.g., “B-C”) in order of increasing velocity:

1. __________ (most negative)
2. __________
3. __________
4. __________
5. __________
6. __________ (most positive)

m. [2] Name the 4 fundamental forces.

1. __________
2. __________
3. __________
4. __________
Problem #2: [10 points total] A German tank fires a shell at Indy, who is trying to hide on a cliff. The shell is fired at an initial angle of $\theta_0 = 25^\circ$. Indy is horizontally $D = 700m$ from the tank. The shell takes 2.8 seconds to reach Indy.

   a. [3] What is the initial speed of the shell?
   b. [4] How high is the cliff?
   c. [3] How what is the speed of the shell when it reaches Indy?

Problem #3: [8 points total] Indiana fires a bullet ($m_1 = 0.05$ kg) right through a thin wooden door ($m_2 = 20$ kg). The initial speed of the bullet is $v_1 = 200m/s$, and the bullet slows down to $v_2 = 50m/s$ after passing through the door.

   a. [5] What is the resulting angular velocity of the door?
   b. [3] How many seconds does it take for the door to rotate $90^\circ$?
Problem #4: [16 points total] To reach the bottom of the Well of Souls, Indy ($m_1 = 70$ kg) ties himself to a boulder ($m_2 = 280$ kg). The rope hangs over a large cylindrical pulley ($m_3 = 30$ kg), but because of friction, the rope does not slip on the pulley. The coefficients of friction between the boulder and the pulley are $\mu_s = 0.3$ and $\mu_k = 0.1$. Indy starts at rest.

a. [3] Determine the maximum value of $\mu_s$ that will even allow Indy to start falling.

b. [3] Since the actual $\mu_s$ is larger than your answer to part (a), Indy puts some rocks in his pockets to make himself heavier. What is the necessary mass of rocks to get Indy started falling?

c. [6] What is his acceleration while he is being lowered (with the rocks in his pockets)?

d. [4] How fast is he going when he reaches the bottom?
Problem #5: [12 points total] Use 6 sig-figs for all calculations in this problem. A dirt-hauling machine used by archeologists contains a flywheel as shown here. It consists of several parts:

ii. The outer rim has a mass $m_1 = 300$ kg and a radial thickness of $r_1 = 0.01$ m.

iii. The inner hub is a cylinder with mass $m_2 = 60$ kg, and a radius of $R_2 = 0.2$ m.

iv. The four spokes each have mass $m_3 = 20$ kg, radius $r_3 = 0.005$ m, and length $L = 0.80$ m.

a. [4] What is the moment of inertia $I_1$ of the rim? *Hint:* the rim is similar to a thin hoop. To account for the radial thickness, use the average value of $R^2$, not the average value of $R$.

b. [2] What is the moment of inertia $I_2$ of the hub?

c. [4] What is the moment of inertia $I_3$ of one spoke about the center of the wheel? Neglect the curvature at the ends of the spokes, but account for the thickness of the spokes.

d. [2] What is the total moment of inertia of the entire flywheel?
Problem #6: [9] In an ancient Mayan temple, Indy translates a legend about the moon moving away from the earth thousands of years ago. Indy knows that currently, the mass of the earth is $m_E = 5.97 \times 10^{24}$ kg, and that the orbital period of the moon is 27.3 days (2358720 s).

a. [3] Currently, how far is the center of the moon from the center of the earth?

b. [3] If the moon were once in orbit twice as close to the earth as the answer to part (a), what must have been its orbital period then?

c. [3] Assuming the moon is a point particle of mass $m_M = 7.35 \times 10^{22}$ kg, what is its current angular momentum with respect to the center of the earth?
Problem #7: [10] While trying to escape from the castle, Indiana Jones and Henry Jones each steal a motorcycle. Unfortunately, they crash into each other in a completely inelastic collision. Each rider has a mass of 70 kg, and each bike has a mass of 330 kg. Initially, Indiana’s velocity is $\mathbf{V}_1 = -20\hat{i} + 35\hat{j}$ km/hr, and Henry’s is $\mathbf{V}_2 = +30\hat{i} + 15\hat{j}$ km/hr.

a. [6] What is the final speed of Indiana after the collision?

b. [4] What fraction of their initial kinetic energy is lost due to the collision?
**Problem #8**: [9 points] Indy is trying to push a large spherical boulder ($m = 1000$kg; $R = 2$m) to unblock a cave entrance. He pushes with a horizontal force of 100N, applied horizontally at a height of $\frac{1}{2}R$ (1.0 m above the ground level). The boulder rolls without slipping.

a. [6] What is the acceleration of the boulder?

b. [3] What is the magnitude of the frictional force exerted on the boulder by the ground?

Extra Credit [1]: You may only receive credit for one of the following. If you attempt multiple questions, I will choose to grade the one you answer most incorrectly.

1. In *Raiders of the Lost Ark*, the staff of Ra was used to find what place?
2. In *Raiders of the Lost Ark*, what artifact was needed to discover the Well of Souls?
3. What state name appears in the title of an “Indiana Jones” movie?
Solid cylinder or disk about the central axis

\[ I = \frac{1}{2} mR^2 \]

Solid cylinder or disk about a central diameter

\[ I = \frac{1}{4} mR^2 + \frac{1}{12} mL^2 \]

Thin rod about an axis through its center, perpendicular to its length

\[ I = \frac{1}{12} mL^2 \]

Solid sphere about any diameter

\[ I = \frac{2}{5} mR^2 \]

Hollow sphere about any diameter

\[ I = \frac{2}{3} mR^2 \]

Thin rod about an axis through an end, perpendicular to its length

\[ I = \frac{1}{3} mL^2 \]