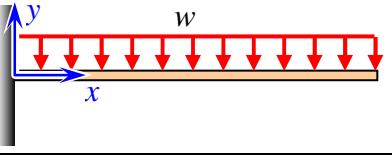
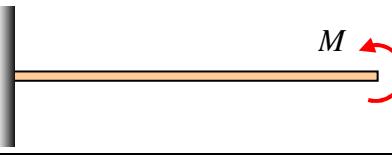
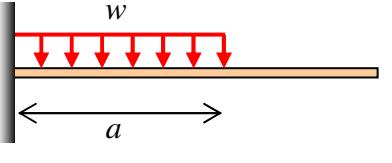
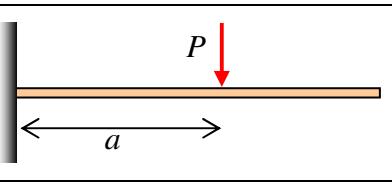
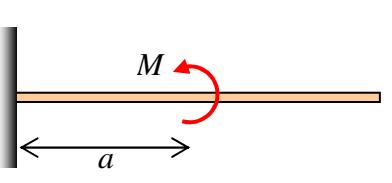
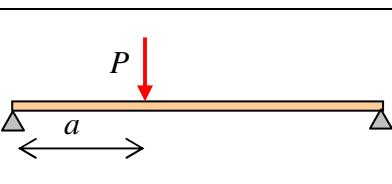
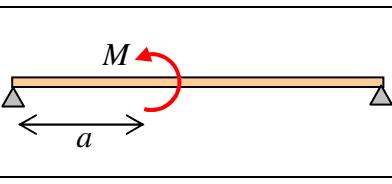
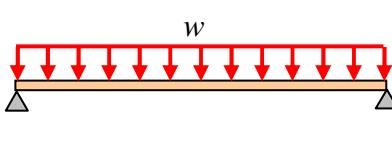
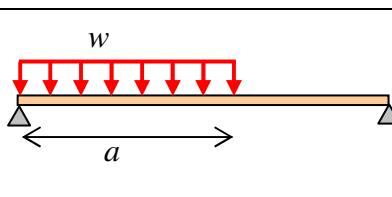


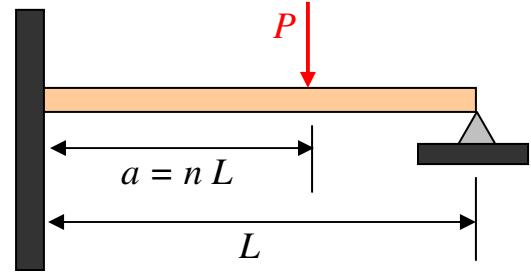
Beam and Loading	Displacement (all beams have total length L)
	$y = -\frac{w}{24EI} (x^4 - 4Lx^3 + 6L^2x^2)$
	$y = \frac{Mx^2}{2EI}$
	$y = \frac{w}{24EI} (4ax^3 - 6a^2x^2 - x^4) + \frac{w}{24EI} (x-a)^4 \langle x-a \rangle^0$
	$y = \frac{P}{6EI} (x^3 - 3ax^2) - \frac{P}{6EI} (x-a)^3 \langle x-a \rangle^0$
	$y = \frac{M}{2EI} x^2 - \frac{M}{2EI} (x-a)^2 \langle x-a \rangle^0$
	$y = \frac{P(L-a)}{6LEI} (x^3 + a^2x - 2aLx) - \frac{P}{6EI} (x-a)^3 \langle x-a \rangle^0$
	$y = \frac{M}{6LEI} (x^3 + 2L^2x - 6aLx + 3a^2x) - \frac{3M}{6EI} (x-a)^2 \langle x-a \rangle^0$
	$y = -\frac{w}{24EI} (x^4 - 2Lx^3 + L^3x)$
	$y = -\frac{wx}{24LEI} (a^4 - 4La^3 + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) + \frac{w}{24EI} (x-a)^4 \langle x-a \rangle^0$

This beam has too many reactions (4 reactions in a 2-D Problem), and 3 of them are unknown (since $R_{Ax} = 0$).

Given P , L , and n , determine the reactions.

We start by defining R_B as some unknown fraction “ q ” of P : $R_B = qP$ (see FBD).

If we can solve for q , then we’ll know everything.



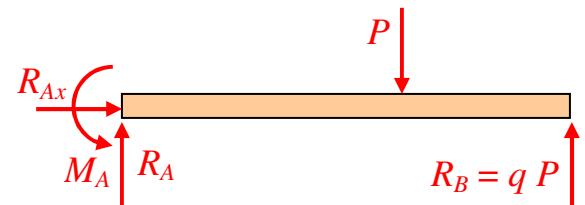
$$\text{N2L: } R_A + R_B = P, \text{ so } R_A = P(1 - q)$$

$$\text{Similarly, N2LR: } M_A = PL(n - q).$$

$$\text{On the left: } V_1(x) = R_A = P(1 - q)$$

$$\text{Integrating: } M_1(x) = P(1 - q)x - M_A, \text{ or}$$

$$M_1(x) = P(1 - q)x - PL(n - q)$$



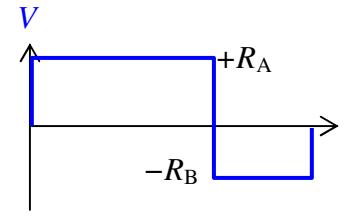
$$\text{On the right: } V_2(x) = -R_B = -qP$$

$$\text{Integrating: } M_2(x) = -qP + R_B L, \text{ or}$$

$$M_2(x) = R_B(L - x) = qP(L - x)$$

These two answers must be consistent: they give the same M at $x = a = nL$.

Next, we use deflections:



$$\text{On the left: } EIy_1'' = M_1$$

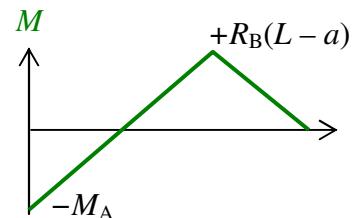
$$EIy_1' = \int M_1 dx = \frac{1}{2}P(1 - q)x^2 - PL(n - q)x + C_2$$

$$\text{BC: } y_1'(0) = 0 \rightarrow C_2 = 0.$$

$$EIy_1 = \int (\dots) dx = (\frac{1}{6})P(1 - q)x^3 - \frac{1}{2}PL(n - q)x^2 + C_4$$

$$\text{BC: } y_1(0) = 0 \rightarrow C_4 = 0.$$

$$EIy_1 = (\frac{1}{6})P(1 - q)x^3 - \frac{1}{2}PL(n - q)x^2$$



$$\text{On the right: } EIy_2'' = M_2$$

$$EIy_2' = \int M_2 dx = qP(Lx - \frac{1}{2}x^2) + C_3$$

$$EIy_2 = \int (\dots) dx = qP(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_3x + C_5$$

$$\text{BC: } y_1'(a) = y_2'(a)$$

$$\text{BC: } y_2(L) = 0$$

$$\rightarrow \text{tons of algebra} \rightarrow C_3 = -\frac{1}{2}PL^2n^2, \text{ and } C_5 = PL^3(\frac{1}{2}n^2 - \frac{1}{3}q)$$

$$\text{BC: } y_1(a) = y_2(a) \rightarrow \text{more algebra}$$

$$q = \frac{1}{2}(3n^2 - n^3)$$

Reactions:

$$R_B = qP$$

$$R_A = (1 - q)P$$

$$M_A = PL(n - q)$$