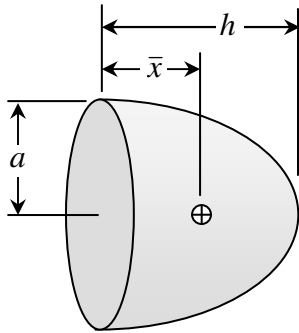
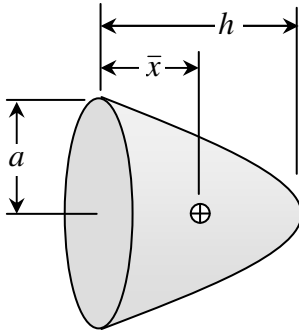
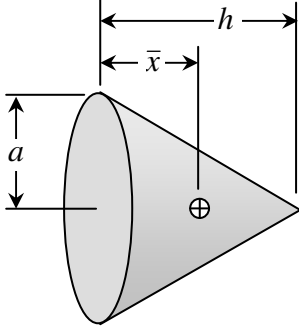
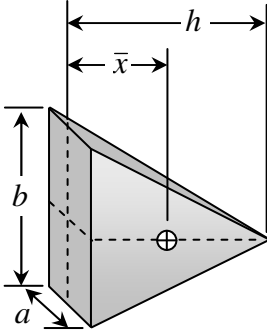
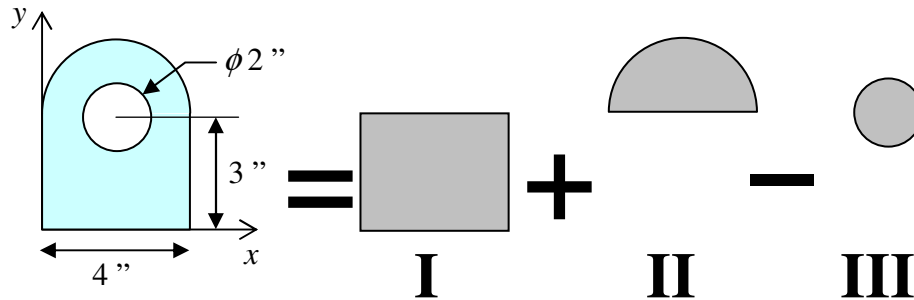


Shape	Sketch	$\bar{x}$	$\bar{y}$	Area
Triangle			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter Ellipse		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Half Ellipse		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Parabola		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Semiparabola		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular Sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

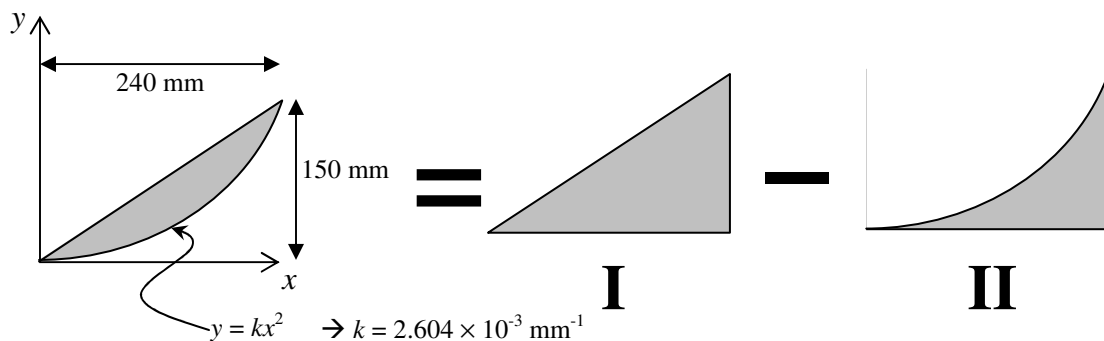
Shape	Sketch	$\bar{x}$	Volume
Semi-ellipsoid		$\frac{3h}{8}$	$\frac{2\pi a^2 h}{3}$
Paraboloid		$\frac{h}{3}$	$\frac{\pi a^2 h}{2}$
Cone		$\frac{h}{4}$	$\frac{\pi a^2 h}{3}$
Pyramid		$\frac{h}{4}$	$\frac{abh}{3}$



Element	Area (in ²)	$\bar{x}$ (in)	$\bar{y}$ (in)	$\bar{x}A$ (in ³)	$\bar{y}A$ (in ³)
I	$(4)(3) = 12.0$	2.0	1.5	24.00	18.00
II	$\frac{1}{2}\pi R^2 = 6.28$	2.0	$3 + (4R)/(3\pi) = 3.85$	12.57	24.18
III	$-\pi R^2 = -3.14$	2.0	3.0	-6.28	-9.42
Σ	+15.14			30.29	32.76

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{30.29}{15.14} = 2.00 \text{ in} \rightarrow \text{We could have guessed this from simple symmetry.}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{32.76}{15.14} = 2.16 \text{ in} \rightarrow \text{We expected it to be less than 2.5 inches or so, so this is believable.}$$



Element	Area (in ²)	$\bar{x}$ (in)	$\bar{y}$ (in)	$\bar{x}A$ (in ³)	$\bar{y}A$ (in ³)
I	$(240)(150)/2 = 18000$	$(2/3)(240) = 160$	$(1/3)(150) = 50$	2.88×10^6	9.0×10^5
II	$-(240)(150)/3 = -12000$	$\frac{3}{4}(240) = 180$	$(3/10)(150) = 45$	-2.16×10^6	-5.4×10^5
Σ	+6000.00			7.2×10^5	3.6×10^5

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{7.2 \times 10^5}{6000} = 120.0 \text{ mm}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{3.6 \times 10^5}{6000} = 60.0 \text{ mm}$$