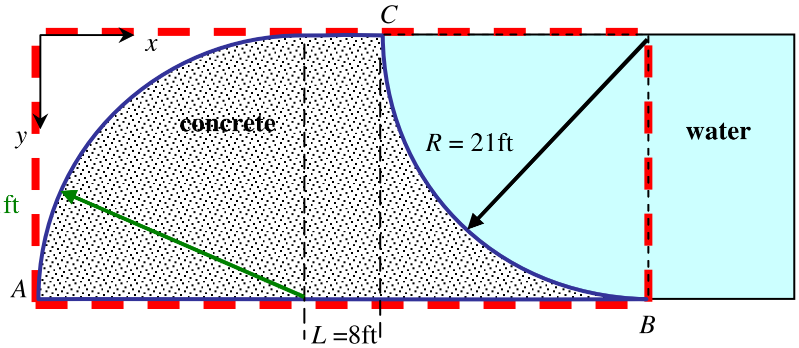
A certain dam is w = 100ft wide (into the page). The cross section is shown. **Find the reactions exerted by the ground on the dam.**

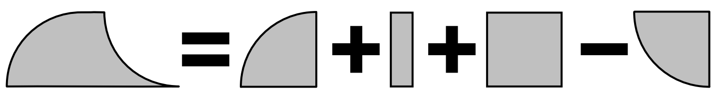


Water: $\gamma_w = \rho_w g$ = 62.4 lb/ft^3
Concrete: $\gamma_c = \rho_c g$ =150 lb/ft^3

My FBD is indicated by the heavy gray dashed line. The forces acting on this body are:

1. The weight of the concrete.
2. The weight of the water enclosed in the FBD
3. Pressure from the water to the right of the FBD
4. Vertical and horizontal reactions from the ground.

1. Weight of the concrete:



Element	Area (ft^2)	$\bar{x}$ (ft)	$\bar{x}A$ (ft^3)
I	346.36	12.087	4186.5
II	168.00	25.00	4200.0
III	441.00	39.50	17419.5
IV	–346.36	41.087	–14230.9
Σ	+609.00		+11575.1

$\bar{x}_{\text{concrete}}$ = (11575.1 ft^3)/(609.0 ft^2) = 19.007 ft

$W_{\text{concrete}} = w\gamma_c A_c$ = (100 ft)(150 lb/ft^3)(609.0 ft^2) = 9,135,000 lb.

2. Weight of the water: The water occupies region "IV" above: A_w = 346.36 ft^2; $\bar{x}_w$ = 41.087 ft.
$W_{\text{water}} = w\gamma_w A_w$ = (100 ft)(62.4 lb/ft^3)(346.36 ft^2) = 2,161,300 lb.

3. Pressure from the right: $F_p = \int P dA = \int_{y=0}^{y=R} (\gamma_w y)(w dy) = \frac{\gamma_w w R^2}{2} = F_p$ = 1,375,900 lb.

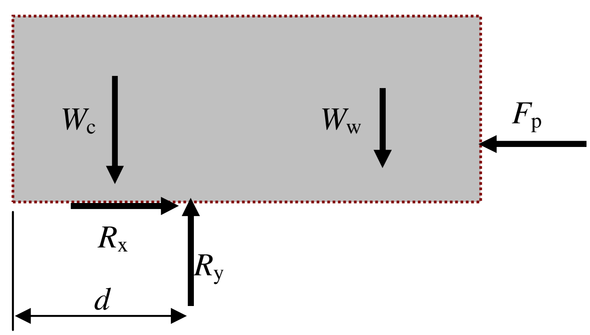
This force acts at $y = (^2/_3)(R)$ = 14 ft.

4. Total FBD:

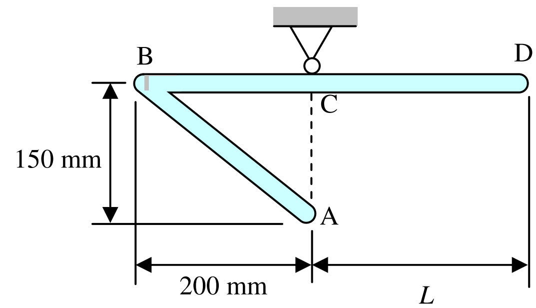
$\Sigma F_x = 0$ → $\boxed{R_x = 1{,}375{,}900 \text{ lb}}$

$\Sigma F_y = 0$ → $\boxed{R_y = 11{,}296{,}300 \text{ lb}}$

$\Sigma M_A = 0$ → $\boxed{d = 22.379 \text{ ft}}$

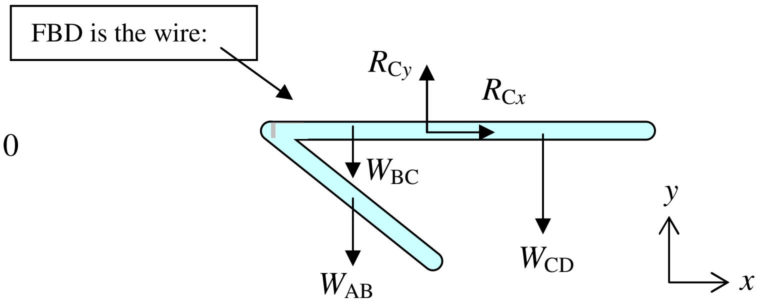


Problem: Determine L to ensure that segment BCD is horizontal when the wire hangs freely.



Let γ= weight per unit length
Pythagorean theorem: L_{AB} = 250 mm

$W_{AB} = \gamma L_{AB}$
$W_{BC} = \gamma L_{BC}$
$W_{CD} = \gamma L$

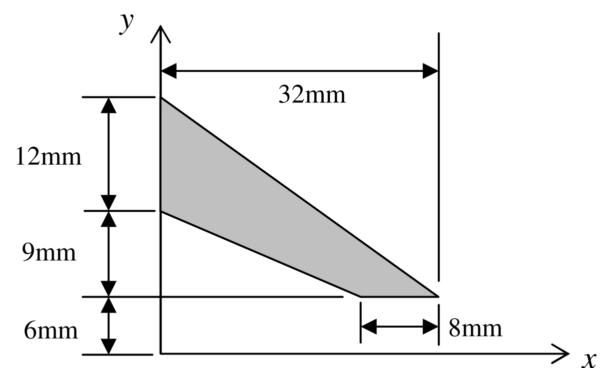


$\Sigma M_C = 0$
$+(W_{BC})(\tfrac{1}{2}L_{BC}) + (W_{AB})(\tfrac{1}{2}L_{BC}) - (W_{CD})(\tfrac{1}{2}L) = 0$

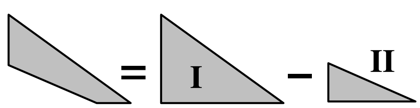
$+\tfrac{1}{2}\gamma L_{BC}^2 + \tfrac{1}{2}\gamma L_{AB}L_{BC} = \tfrac{1}{2}\gamma L^2$

$\rightarrow \boxed{L = 300 \text{ mm}}$

Find the volume and mass of the iron collar:
Given: ρ_{iron} = 7200 kg/m^3.

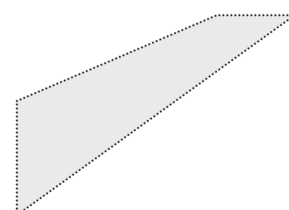


Step I: Find $\overline{Y}$. The shaded shape is composed of two triangles:



Element	Area (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
I	336	6 + 21/3 = 13	4368
II	–108	6 + 9/3 = 9	–972
Σ	228		+3396

$$\overline{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{3396 \text{ mm}^3}{228 \text{ mm}^2} = \overline{Y} = 14.895 \text{ mm}$$



$V = 2\pi\overline{Y}A = (2\pi)(14.895 \text{ mm})(228 \text{ mm}^2) = \boxed{V = 21338 \text{ mm}^3}$

$m = \rho V = (7200 \text{ kg/m}^3)(21338 \text{ mm}^3)\times(1\text{m}/1000\text{mm})^3 \times(1000\text{g}/1\text{kg}) = \boxed{m = 153.6 \text{ g}}$