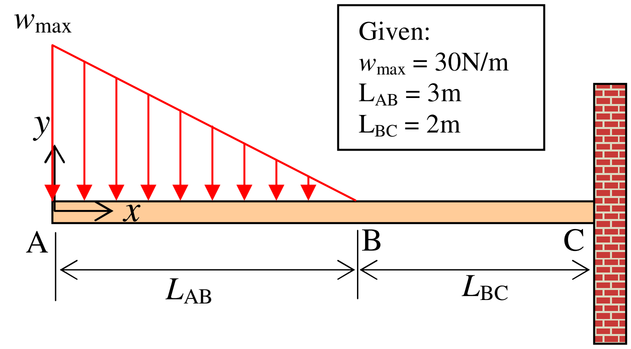
Find the shear force and bending moments in the beam having the loading shown.



First, find $w_{AB}(x)$: $w_{AB}(x) = w_{max}(1 - x/L_{AB})$

Find the shear force for section AB:

$$V_{AB}(x) = -\int w_{max}\left(1 - \frac{x}{L_{AB}}\right)dx$$

$$V_{AB}(x) = -w_{max}\left(x - \frac{x^2}{2L_{AB}}\right) + C_1 \quad \text{but } C_1 = 0\text{, so}$$

$$\boxed{V_{AB}(x) = -w_{max}\left(x - \frac{x^2}{2L_{AB}}\right)}$$

Find the V_B to use as a boundary condition for the next region:

$$V_B = -w_{max}\left(L_{AB} - \frac{L_{AB}^{\;2}}{2L_{AB}}\right) = \frac{-w_{max}L_{AB}}{2} \;\rightarrow\; \boxed{V_B = -45\text{N}}$$

Find the shear force for section BC. In this region, $w = 0$:

$V_{BC}(x) = -\int 0dx = C_2 = V_B$ because if V_{BC} is constant, then it must be the same as point B which we already know.

$$\boxed{V_{BC}(x) = \frac{-w_{max}L_{AB}}{2}}$$

Find the bending moment for section AB:

$$M_{AB}(x) = \int -w_{max}\left(x - \frac{x^2}{2L_{AB}}\right)dx = -w_{max}\left(\frac{x^2}{2} - \frac{x^3}{6L_{AB}}\right) + C_3 \qquad \text{But, } M \text{ at point A is zero, so } C_3 = 0\text{:}$$

$$\boxed{M_{AB}(x) = -w_{max}\left(\frac{x^2}{2} - \frac{x^3}{6L_{AB}}\right)}$$

Find the M_B to use as a boundary for the next region: $M_B = -w_{max}\left(\frac{L_{AB}^2}{2} - \frac{L_{AB}^{\;3}}{6L_{AB}}\right) = \frac{-w_{max}L_{AB}^{\;2}}{3} \rightarrow \boxed{M_B = -90\text{Nm}}$

Finding the bending moment for section BC:

$$M_{BC}(x) = \int\left(\frac{-w_{max}L_{AB}}{2}\right)dx = \frac{-w_{max}L_{AB}x}{2} + C_3 \qquad \text{Using } M_B \text{ at } x = L_{AB}\text{:}$$

$$\frac{-w_{max}L_{AB}^{\;2}}{3} = \frac{-w_{max}L_{AB}L_{AB}}{2} + C_3 \quad \rightarrow \quad C_3 = \frac{+w_{max}L_{AB}^{\;2}}{6}$$

$$M_{BC}(x) = \frac{-w_{max}L_{AB}x}{2} + \frac{w_{max}L_{AB}^{\;2}}{6} \qquad \boxed{M_{BC}(x) = \frac{w_{max}L_{AB}}{6}\left(L_{AB} - 3x\right)}$$

The shear and bending moment diagrams are shown: