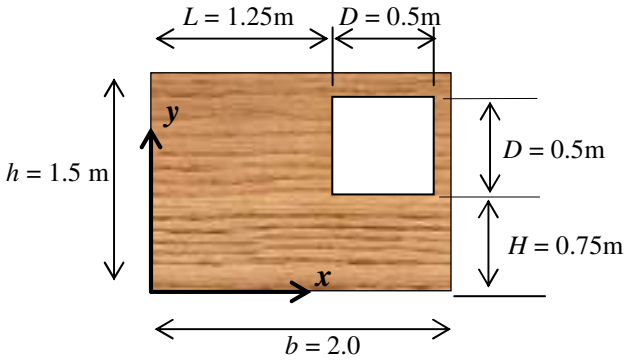
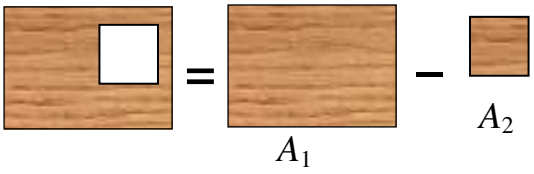
2D Center of Mass Example Problem

While building a tree house for his kids, dad builds a wall by starting with a large 1.5×2.0 m piece of plywood. He then cuts a hole for a window. The hole is 0.50×0.50 m square, and located as shown. To make sure he attaches it to the tree without having it be off-balance, he wants you to determine the center of mass of this wall.





Step 1: Identify the parts (see sketch above).
Step 2: Notice that for any part, the mass is proportional to the area. So, instead of using units of kg, we can use area (m^2) for the masses of the parts.
Step 3: Compute areas and positions:

$A_1 = bh = 3.0\ \text{m}^2$
$A_2 = -D^2 = -0.25\ \text{m}^2$ (because this represents removed plywood, this area is *negative*!)
$A_{\text{total}} = A_1 + A_2 = 2.75\ \text{m}^2$
$x_{\text{cm1}} = b/2 = 1.0$ m $\qquad y_{\text{cm1}} = h/2 = 0.75$ m
$x_{\text{cm2}} = L + \tfrac{1}{2}D = 1.50$ m $\qquad y_{\text{cm2}} = H + \tfrac{1}{2}D = 1.00$ m

$$\text{So } x_{CM} = \frac{1}{2.75\text{m}^2}\left[(1\text{m})(3\text{m}^2) + (1.50\text{m})(-0.25\text{m}^2)\right] = \boxed{x_{\text{CM}} = 0.955\ \text{m}}$$

$$\text{Also, } y_{CM} = \frac{1}{2.75\text{m}^2}\left[(0.75\text{m})(3\text{m}^2) + (1.00\text{m})(-0.25\text{m}^2)\right] = \boxed{y_{\text{CM}} = 0.727\ \text{m}}$$