Math 160, Assignment #1 Assigned Nov 15, 2021
Due On Nov 29, 2021

Name: ________________________

Textbook Sections that might be helpful: Chapter 13, and a little from Chapter 11.
You are expected to use Excel for many of the calculations in this assignment.
Organize your results professionally! I haven't yet decided the point value for each problem, but you can assume that the harder it is, the more it is worth.

1. A used car salesman has 58 cars in his lot. He checks the tire pressure in all 4 tires of 7 of his cars and discovers that the pressure seems to follow a normal distribution. Specifically, he measures the following values, all in "psi" (this data is also available in an Excel document on the course homepage):

35.3	37.6	41.2	34.2	37.7	30.8	39.4
33.1	39.3	26.4	35.2	38.0	30.0	35.9
31.8	35.1	45.4	30.1	31.2	32.2	34.5
35.1	32.7	40.6	32.8	30.7	32.8	38.6

a. Determine the mean μ, median, standard deviation σ, and uncertainty in the mean σ_m for these 28 tire pressures.

Notation: When you write the standard deviation, you must use exactly 2 significant digits. Using 1 is wrong. Using 3 is wrong. Significant digits is not the same thing as decimals. If you don't remember what it is, look it up!
Then, when you write the mean, you must write it so that it ends up having the same number of decimal places as the standard deviation. In these examples, each arrow points to the part that is wrong...

Examples:	Correct	Correct	Incorrect	Incorrect	Incorrect
σ	3.2	46	86.22 ←	0.7 ←	7.2
μ	12.8	877	341.88	18.4	25.34 ←

b. Knowing that tire pressure is supposed to be at least 30 psi, estimate the total number of *tires* (i.e., using all 58 cars) for which you expect the pressure to be too low.

---

2. Using μ = \$150 and σ = \$35, make a plot in Excel of the associated Normal density function for the price of used Math Textbooks. Your goal is NOT to be "creative", it is to be professional. Your horizontal axis should range from \$0 to \$250. Your grade will depend on line thickness, color, aspect ratio, clarity of labels, font size, etc. When you print it to submit it, your plot should be around 3 inches tall, and about an inch wider than it is tall. Don't print anything until you are ready to print everything for the entire assignment... that way you can do all your printing on one sheet.

Reminders: $\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\qquad \sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\mu)^2}$ $\qquad f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Excel tip: to find e^3, type `= exp(3)`

---

3. On the same Excel document that you found for Problem 1, you'll find some data about 2021 Olympic Men's Marathon times.

a. Compute the minimum, maximum, total, μ, σ, and median for these 76 values.

b. Now let's make a histogram: The numbers are given in alphabetical order by runner, so sort them by time from smallest to largest.

c. In some new place, make a list of 9 sizes, called "bins". Each bin is 4 minutes "wide". Then next to this column, make another column counting the number of runners having times that correspond to this bin:

Bin: Time in Minutes	**Number of Runners having a time that is more than this number, but less than the next number**
124 to 128	(just count these by hand!)
128 to 132	
132 to 136	Example: I got 16 for this.
136 to 140	
140 to 144	
144 to 148	
148 to 152	
152 to 156	
156 to 160	

d. In Excel, select the last column only, including it's label (i.e., all 9 rows). Then insert a "column" or "bar" chart. Right click the plot itself. Choose "Select Data". Below "Horizontal Axis Labels", click "Edit". Click in the "Axis Label Range" once, then highlight the first column of your data on the main spreadsheet (i.e., your 9 bin names). Make the plot look great with professional fonts, line widths, colors, etc. Again, the final size is maybe 3 inches tall by 4 inches wide. You'll be printing this out along with the printout from Problem 2. If I were you, I'd past the two plots into Word or something before printing them, so you don't clutter the printout with extraneous stuff.

---

4. Let's make a Normal density function f(x). If "A" = 1, "B" = 2, etc, find the largest number corresponding to any letter in your own first or last name, as you wrote it above. Since my name has a "Z" in it, I would use 26. Use this number for μ. Record the letter and the number. Then, ignoring the letters A, B, C, D, and E, determine the remaining *smallest* number in your own name. Since my name has no "F", but has a "G" in it, I would use 7. Use this number for σ.

a. Then determine and record the value of f(x) for the following 5 values of x:

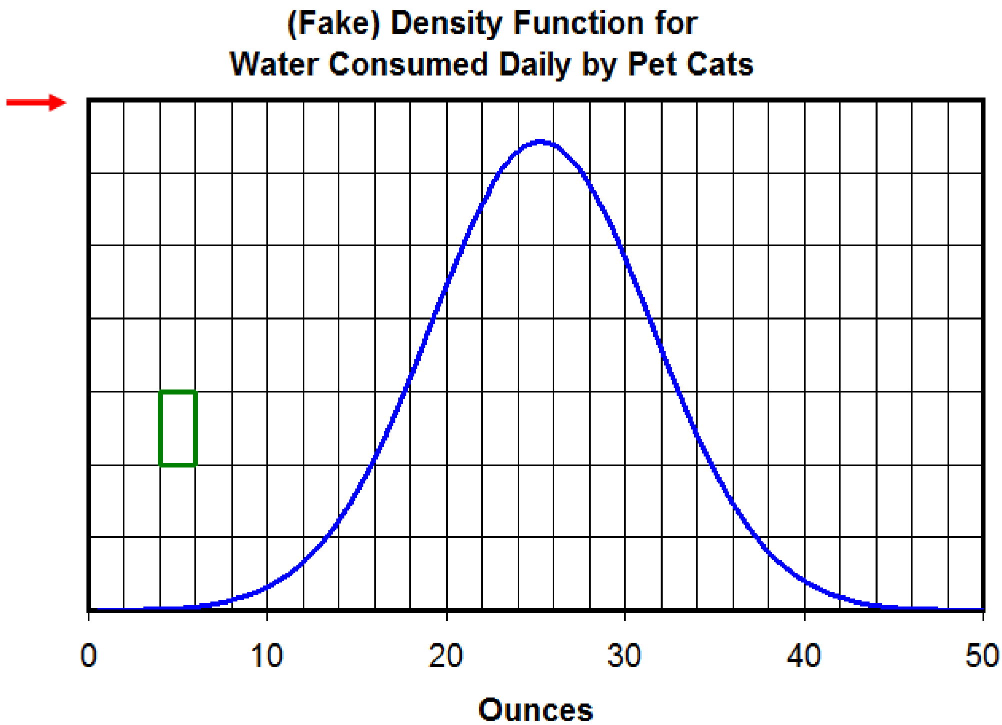
$x = \mu - 2\sigma$, $x = \mu - 1\sigma$, $x = \mu - 0\sigma$, $x = \mu + 1\sigma$, $x = \mu + 2\sigma$

b. Find the ratio $\dfrac{f(\mu+\sigma)}{f(\mu)}$.

c. Find the ratio $\dfrac{f(\mu+2\sigma)}{f(\mu)}$. You will use these values in the next problem!

Interesting note: These ratios come out to the same values no matter what numbers you use for μ and σ!

5. Here's a Normal density function. You'll be making measurements and drawing on this plot. So, use a pencil, not a pen, and don't push too hard (in case you need to erase).



a. See the green rectangle? Let's call it a “block”. How wide it, side to side? Include units.

b. Estimate μ using a ruler. Hint: it is not an integer! You should get it right within 2% of the true answer.

c. In “blocks”, about how tall is this function? Do not use an integer.

d. Now measure the actual height of this function in cm with a ruler. Then, multiply this measurement by your answer to problem 4b to find a slightly smaller height, still in cm. Draw a horizontal line at this new height. Note the two values of "x" (i.e., in ounces) where your new line intersects the blue density function. Use a ruler to draw vertical lines downwards from these two intersections. Label the leftmost one x_1 and the rightmost one x_2. What are the values of x_1 and x_2?

e. You now have three ways to estimate the standard deviation. Do them all. One is $(x_2 - \mu)$, another is $(\mu - x_1)$, and the third is $\frac{1}{2}(x_2 - x_1)$. Which of them is the best estimate, and why?
f. By direct counting, estimate the number of "blocks" that lie under the blue curve. Do your best to account for half-blocks and so on. Record it to the nearest integer, for example "total blocks = 72".

This plot doesn't directly show probability. Instead, the probability is based on area under the curve. For example, the probability that a cat drinks between 30 and 40 ounces of water each day is equal to the number of blocks under the plot between 30 and 40, divided by the total number of blocks.

g. Estimate the following probabilities:

i. the probability that a cat drinks between 30 and 40 ounces of water each day.
ii. the probability that a cat drinks between 0 and 50 ounces of water each day.
iii. the probability that a cat drinks between μ and $(\mu + \sigma)$ ounces of water
iv. the probability that a cat drinks between $(\mu - \sigma)$ and $(\mu + \sigma)$ ounces of water
v. the probability that a cat drinks between $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$ ounces of water
vi. the probability that a cat drinks between (μ) and 50 ounces of water each day.

Hint: Except for part i, the rest of these can be done without actually counting blocks, since they are answered directly in the textbook as being common results that are true for ALL Normal density functions!

h. You already found the total area in “blocks”. But nobody really uses “blocks”. Instead, we think of the area under the curve as being a pure number (that is, it has no units). Since the area of a block is base × height, and we already found that the base DID have units in part a, what must the units of “height” be so that the resulting area has no units? Yes, the answer is a little strange!

j. Since your answer to part g_{ii} was 1.00 (or 100%), then:

$$(\text{1 block base}) \times (\text{1 block height}) \times (\text{total number of blocks}) = 1.00$$

You already found a value for (1 block base) in part a, and (total number of blocks) in part f. Now, solve this expression for the height of each block. Hint: it's a pretty round number. But, the answer will use the strange units you discovered in part h... don't forget to write that as part of your answer!

k. Use your answers to parts c and j to determine how tall this function is in proper units.

m. The boundary (or frame) of the plot is 7 blocks tall. What is the height of the frame of the plot (in other words, what is the value that the red arrow points to)? Again, it will have the same weird units as parts h, j, and k!