Math Methods, Final Exam Equation Sheet — Dr. Pogo

Spring 2023

Series

$$f(x+h) = f(x) + hf'(x) + \frac{hf''(x)}{2!} + \frac{hf'''(x)}{3!} + \ldots$$

$$\sum_{i=0}^{n} ar^i = \frac{a(1-r^{n+1})}{1-r} \qquad \sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$$

$$\sum_{i=1}^{n} ar^i = \sum_{i=0}^{n} ar^i - a \qquad \sum_{i=1}^{\infty} ar^i = \frac{ar}{1-r}$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n =$$

$$= 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \ldots$$

Coordinate Systems

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$dA = rdrd\theta$$
$$ds = \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}\,dr$$

$$dV = rdrd\theta dz$$
$$dA = rd\theta dz$$
$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$$

$$x = r\sin(\theta)\cos(\phi)$$
$$y = r\sin(\theta)\sin(\phi)$$
$$z = r\cos(\theta)$$
$$dV = r^2\sin(\theta)drd\theta d\phi$$
$$dA = r^2\sin(\theta)d\theta d\phi$$
$$ds^2 = dr^2 + r^2 d\theta^2 + r^2\sin^2(\theta)d\phi^2$$

ODE's

$$y' + Py = Q \rightarrow$$
$$I = \int P dx$$
$$y = e^{-I}\int Qe^{I}dx + ce^{-I}$$

Given r_1, r_2:

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad (r_1 \neq r_2)$$
$$y = (Ax+B)e^{r_1 x} \quad (r_1 = r_2)$$
$$y = Ae^{r_1 x} + Be^{r_2 x} \quad (r_1, r_2 = \alpha \pm i\beta)$$

or $y = e^{\alpha x}\left(A\sin(\beta x) + B\cos(\beta x)\right)$

or $y = Ce^{\alpha x}\sin(\beta x + \gamma)$

$$y_p = \begin{cases} Ce^{cx} & \text{if } c \neq a,\ c \neq b \\ Cxe^{cx} & \text{if } c = (a \text{ or } b),\ a \neq b \\ Cx^3e^{cx} & \text{if } c = a = b \end{cases}$$

Probability

$$p(AB) = p(A)p_A(B)$$
$$p(A+B) = p(A) + p(B) - p(AB)$$
$$P(n,r) = \frac{n!}{(n-r)!}$$
$$C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$
$$\sigma^2 = \sum_i (x_i - \mu)^2 f(x_i)$$
$$= \int_{-\infty}^{\infty} \sum_i (x-\mu)^2 f(x)dx$$

Binomial: $f(x) = C(n,x)p^x q^{n-x}$

Normal: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$$\sigma^2 \approx \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2$$
$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

Poisson: $P_n = \frac{\mu^n}{n!}e^{-\mu}$

Inertia Tensors

$$I_{xx} = \sum_i m_i\left(y_i^2 + z_i^2\right) = \int\left(y^2 + z^2\right)dm$$
$$I_{xy} = -\sum m_i x_i y_i = -\int xydm$$

Fourier

$$f(x) = \frac{1}{2}a_0 + a_1\cos\left(\frac{1\pi x}{l}\right) + a_2\cos\left(\frac{2\pi x}{l}\right) + \ldots$$
$$+ b_1\sin\left(\frac{1\pi x}{l}\right) + b_2\sin\left(\frac{2\pi x}{l}\right) + \ldots$$
$$a_n = \frac{1}{l}\int_{-l}^{+l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$
$$b_n = \frac{1}{l}\int_{-l}^{+l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$
$$f(x) = \int_{-\infty}^{+\infty} g(\alpha)e^{i\alpha x}d\alpha$$
$$g(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(x)e^{-i\alpha x}dx$$

Derivatives & Integrals

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \ldots$$
$$ds = \sqrt{dx^2 + dy^2}$$
$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$
$$\bar{x} = \frac{\int xdm}{\int dm}$$
$$I = \int r^2 dm$$

Complex Numbers

$$z = x + iy = re^{i\theta}$$
$$z = r\cos(\theta) + ri\sin(\theta)$$
$$\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$
$$\sinh(z) = \frac{e^{z} - e^{-z}}{2}$$
$$\cosh(z) = \frac{e^{z} + e^{-z}}{2}$$

Linear Algebra

Active Rotation Matrix : $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Passive Rotation Matrix : $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Vectors

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$
$$\nabla^2\phi = \vec{\nabla}\cdot\vec{\nabla}\phi$$

$$\vec{V} = \vec{\nabla}\phi$$
$$\psi = \int v_x dy = -\int v_y dx$$